Jet evolution in a dense QCD medium: II

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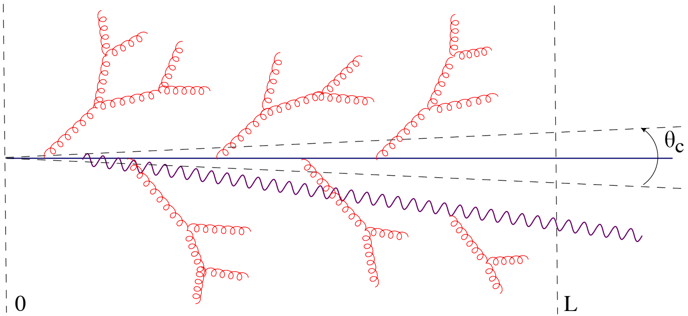
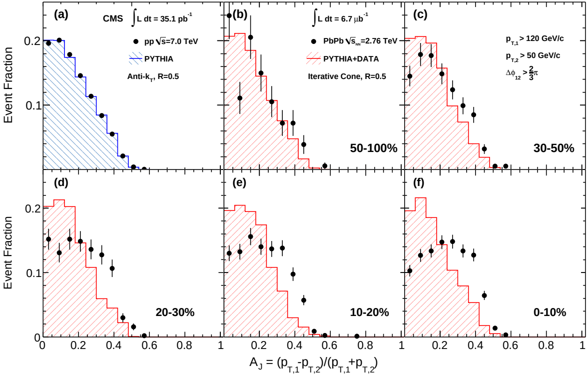


Di–jet asymmetry : A_J (CMS)

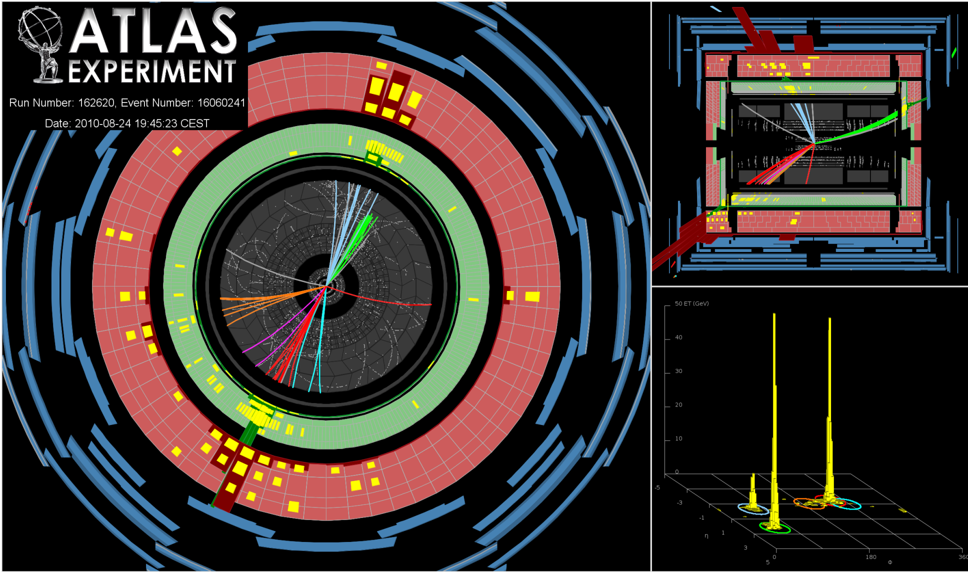


- Event fraction as a function of the di–jet energy imbalance in p+p (a) and Pb+Pb (b–f) collisions for different bins of centrality

$$A_J = \frac{E_1 - E_2}{E_1 + E_2} \qquad (E_i \equiv p_{T,i} = \text{ jet energies})$$

- N.B. A pronounced asymmetry already in the p+p collisions !

Tri–jets in p+p collisions at the LHC



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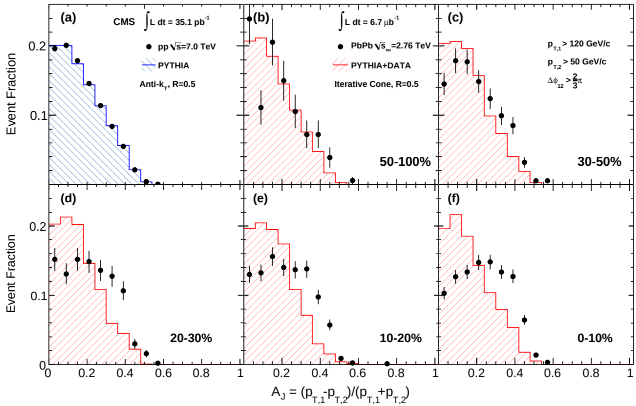
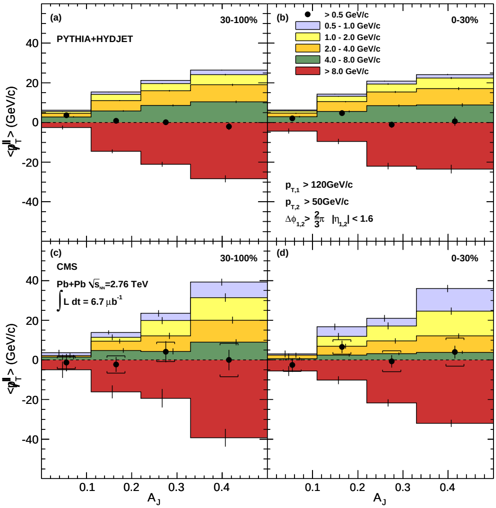


- Additional energy imbalance as compared to p+p : 20 to 30 GeV
- Considerably larger than the typical scale in the medium: the 'temperature' $T \sim 1$ GeV (average $p_\perp$)

No missing energy ! *(CMS, arXiv:1102.1957)*

- ... but a pronounced difference in its distribution in bins of $\omega \equiv p_T$

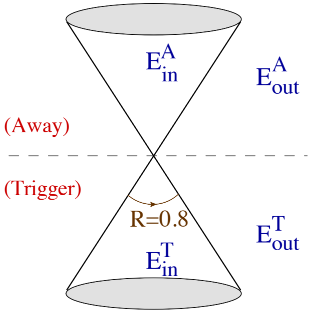
- $p_T^{\|}$: projection of a hadron energy along the jet axis
- $p_T^{\|} < 0$: same hemisphere as the trigger jet
- $p_T^{\|} > 0$: same hemisphere as the away jet
- all hadrons with $p_T > 0.5$ GeV are measured

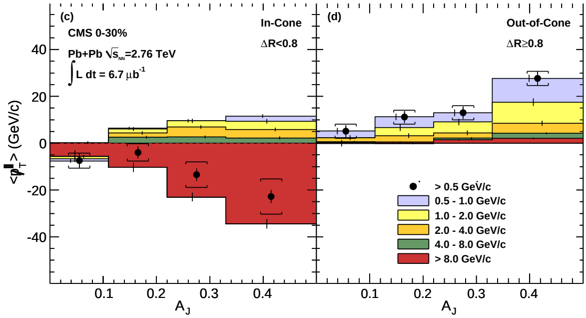


- Pb+Pb: excess of soft hadrons (≤ 2 GeV) in the 'away' hemisphere

These soft hadrons are found at large angles

- The energy imbalance for a jet with a wide opening : $R = 0.8$



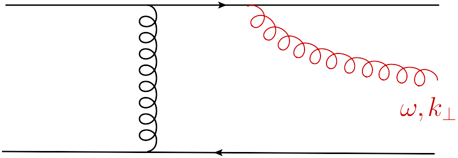


- Di–jet asymmetry : $E^T_{in} > E^A_{in}$
- No missing energy : $E^T_{in} + E^T_{out} = E^A_{in} + E^A_{out}$
- The energy lost at large angles, $E^A_{out} - E^T_{out}$ ...

... is carried mostly by soft hadrons with $p_T < 2$ GeV

Bremsstrahlung

- After a hard scattering, a charged parton radiates a gluon



- Bremsstrahlung favors the emission of soft & collinear radiation

$$\mathrm{d}P = \frac{\alpha_s C_R}{\pi^2}\,\frac{\mathrm{d}\omega}{\omega}\,\frac{\mathrm{d}^2\boldsymbol{k}}{\boldsymbol{k}^2} \qquad \text{(probability density)}$$

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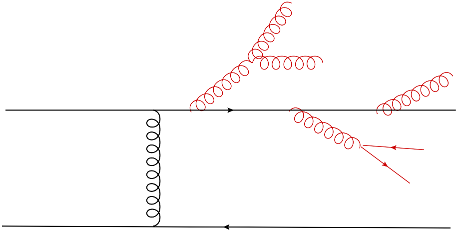

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- Multiple emissions ('fragmentation') leading to a jet structure

Bremsstrahlung

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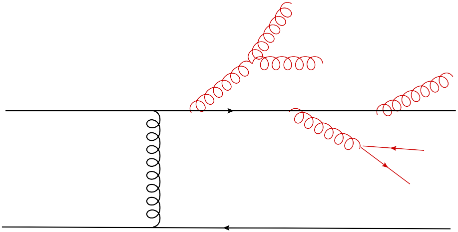


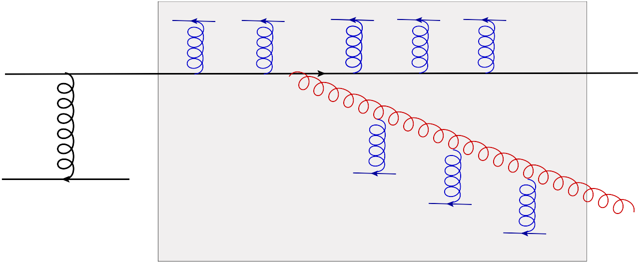
- Bremsstrahlung favors the emission of soft & collinear radiation

$$\Delta P = \frac{\alpha_s C_R}{\pi^2} \ln \frac{E}{\omega} \ln \frac{Q^2}{\boldsymbol{k}^2}$$

- Multiple emissions ('fragmentation') leading to a jet structure
- Each additional suppression is suppressed by a factor $\alpha_s C_R$, but enhanced by the (energy & transverse momentum) phase–space

Medium–induced radiation

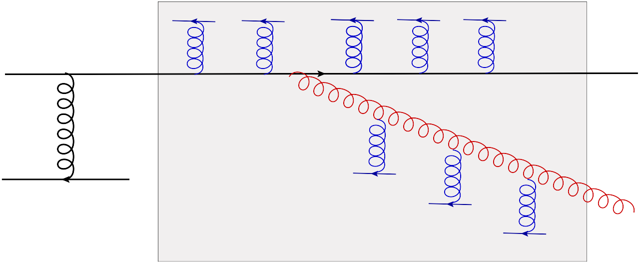
- AA collisions : the partons produced by a hard scattering can further interact with the medium constituents



- Such interactions trigger additional radiation as compared to that that would be produced by an off–shell parton in the vacuum
- This medium-induced radiation has distinguished characteristics:
  - a different emission probability (or 'gluon spectrum')
  - a different angular distribution
  - a different fragmentation pattern

The BDMPSZ mechanism

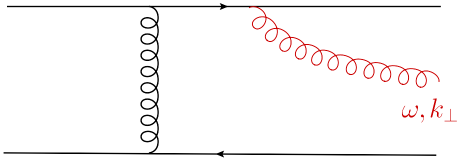
- pQCD description originally developed for a single gluon emission
  (Baier, Dokshitzer, Mueller, Peigné, and Schiff; Zakharov, 96–97)
- Gluon emission is linked to transverse momentum broadening



- Appropriate for the total energy loss by the leading particle (e.g. R_{AA})
- The LHC data call for a global understanding of the jet evolution
- Recent extension of the theory to multiple medium–induced emissions
  (Blaizot, Dominguez, E.I., Mehtar-Tani, 2012–13)

The formation time (1)

- In–medium rescattering changes the mechanism for gluon emission
  - in the vacuum: the virtuality Q^2 of the parent parton



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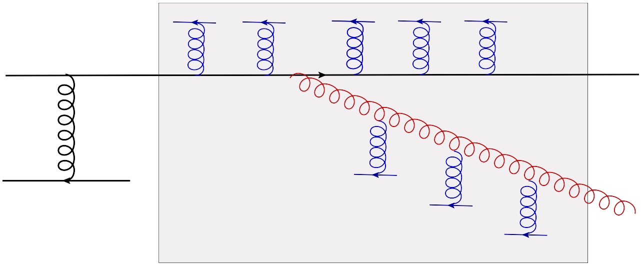
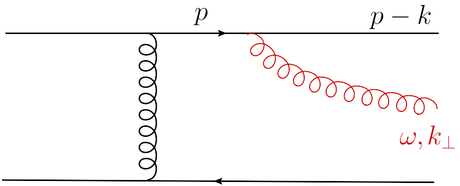


  - in the medium: the acceleration provided by the transverse kicks
- This leads to different formation times & spectra
- Remember: in QM, it takes some time to emit a gluon (or photon)

The formation time (2)

- The formation time can be estimated from the uncertainty principle :

$$\tau_f = \frac{1}{\Delta E} = \frac{1}{\omega + E_{p-k} - E_p}$$



  - $p^\mu = (p_0, 0, 0, p_z)$, $Q^2 = p_0^2 - p_z^2 > 0$, $E_p = p_z$;
  - $k^\mu = (\omega, \boldsymbol{k}, k_z)$, $\omega = \sqrt{k_z^2 + \boldsymbol{k}^2} \simeq k_z + \frac{k_\perp^2}{2k_z}$;
  - $E_{p-k} = \sqrt{(p_z - k_z)^2 + \boldsymbol{k}^2} \simeq p_z - k_z + \frac{k_\perp^2}{2(p_z - k_z)}$

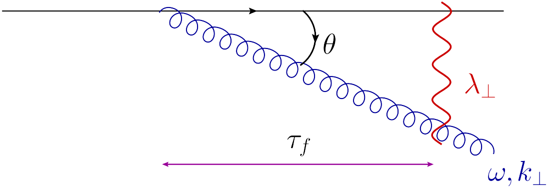
- Soft gluon : $k_z \ll p_z \implies \tau_f \simeq \frac{2k_z}{k_\perp^2}$

The formation time (3)

- A more physical argument : quantum decoherence
- The gluon must lose quantum coherence with respect to its source

  ▷ the quark–gluon transverse separation $b_\perp$ at the formation time τ_f must be larger than the gluon transverse wavelength $\lambda_\perp = 2/k_\perp$

- High energy kinematics: $\omega \gg k_\perp \implies$ small angle: $\theta \simeq k_\perp/\omega$

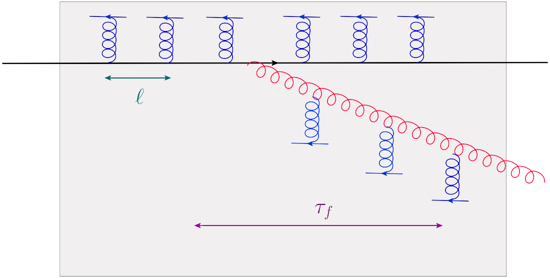


$$b_\perp \simeq \theta\,\tau_f \gtrsim \lambda_\perp \simeq 2/k_\perp \implies \tau_f \simeq \frac{2}{\omega\theta^2} \simeq \frac{2\omega}{k_\perp^2}$$

- In light–cone coordinates: $\tau_f \simeq \frac{2k^+}{k_\perp^2} = 1/k^-$

Formation time in the medium

- The formation time is controlled by the gluon kinematics: $\tau_f \simeq \frac{\omega}{k_\perp^2}$
  - in the vacuum: the kinematics is fixed at the emission vertex
  - in the medium: it is further altered by scattering during formation



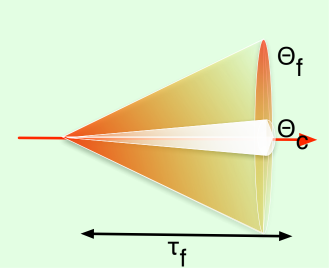
  - a lower limit on the gluon transverse momentum: $\Delta k_\perp^2 \sim \hat{q}\tau_f$
  - an upper limit on the formation time :

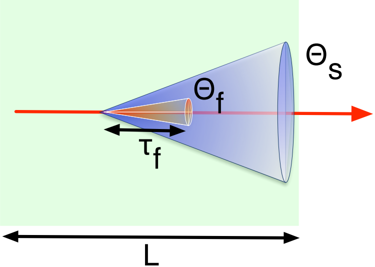
$$\tau_f = \frac{2\omega}{k_\perp^2} \quad \& \quad k_\perp^2 \gtrsim \hat{q}\tau_f \quad \Longrightarrow \quad \tau_f \lesssim \sqrt{\frac{2\omega}{\hat{q}}}$$

Formation time & emission angle

$$\tau_f(\omega) \simeq \sqrt{\frac{2\omega}{\hat{q}}} \quad \& \quad \theta_f(\omega) \simeq \frac{\sqrt{\hat{q}\tau_f}}{\omega} \simeq \left(\frac{2\hat{q}}{\omega^3}\right)^{1/4}$$

- Soft gluons (small ω) : short formation times & large emission angles





- Maximal ω for this mechanism : $\tau_f \simeq L \Rightarrow \omega_c = \hat{q}L^2/2$
- Minimal emission angle: $\theta_c \equiv \theta_f(\omega_c) \simeq 2/\sqrt{\hat{q}L^3}$
- Soft gluons $(\omega \ll \omega_c)$ have $\tau_f \ll L$ & $\theta_f \gg \theta_c$

Formation time & emission angle

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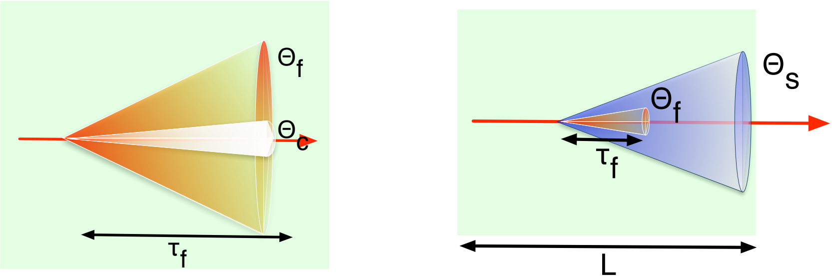

- After emission, $k_\perp$ can further increase, up to a final value $\sim Q_s$

$$Q_s^2 = \hat{q}L \gg \hat{q}\tau_f(\omega) \implies \theta_s \simeq \frac{Q_s}{\omega} \gg \theta_f$$

Formation time & emission angle

$$\tau_f(\omega) \simeq \sqrt{\frac{2\omega}{\hat{q}}} \quad \& \quad \theta_f(\omega) \simeq \frac{\sqrt{\hat{q}\tau_f}}{\omega} \simeq \left(\frac{2\hat{q}}{\omega^3}\right)^{1/4}$$

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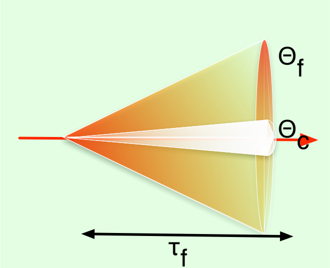
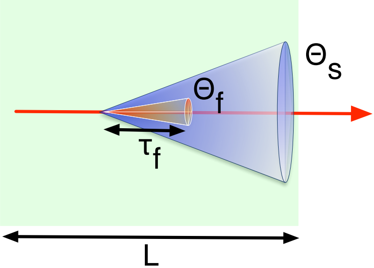




- Typical values (consistent with the phenomenology) :

$\hat{q} \simeq (1 \div 2)$ GeV^2/fm, $L \simeq 4$ fm, $\omega_c \simeq 40$ GeV, $Q_s \simeq 2$ GeV, $\theta_c \simeq 0.1$

LPM effect *(Landau, Pomeranchuk, Migdal, within QED)*

- The probability density for an emission with energy ω (and any $k_\perp$)

$$\frac{\mathrm{d}P}{\mathrm{d}\omega} \simeq \frac{\alpha_s}{\omega}\,\frac{L}{\tau_f(\omega)}$$ 'BDMPSZ spectrum'

- Bremsstrahlung $\times$ average number of emissions :
  - the gluon can be emitted anywhere inside the medium (L), but its emission takes a typical time $\tau_f(\omega)$
- The BDMPSZ regime corresponds to $L \gtrsim \tau_f(\omega) \gg \ell$
  - a large number of successive collisions, of order $\tau_f(\omega)/\ell$, which coherently contributes to a single emission
  - suppression factor $\ell/\tau_f(\omega) \ll 1$ as compared to incoherent emissions (LPM suppression)
- The emission rate rapidly decreases with ω : $\mathrm{d}P/\mathrm{d}\omega \propto 1/\omega^{3/2}$

The average energy loss

$$\omega \frac{\mathrm{d}P}{\mathrm{d}\omega} \simeq \alpha_s \frac{L}{\tau_f(\omega)} \simeq \alpha_s \sqrt{\frac{\omega_c}{\omega}} \qquad (\omega < \omega_c \equiv \hat{q}L^2/2)$$

- The average energy loss by a leading particle with energy $E > \omega_c$

$$\Delta E = \int^{\omega_c} \mathrm{d}\omega\, \omega \frac{\mathrm{d}P}{\mathrm{d}\omega} \sim \alpha_s \omega_c \sim \alpha_s \hat{q} L^2$$

  - integral dominated by its upper limit $\omega = \omega_c$

- Hard emissions with $\omega \sim \omega_c$: probability of $\mathcal{O}(\alpha_s)$
  - rare events but which take away a large energy
  - energy loss ω_c per event $\Longrightarrow$ average energy loss $\Delta E \sim \alpha_s \omega_c$
  - small emission angle $\theta_c \Rightarrow$ the energy remains inside the jet

- Irrelevant for the di–jet asymmetry ☹

Soft emissions at large angles

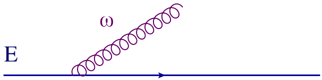
- Recall: Soft gluons $(\omega \ll \omega_c)$ have $\tau_f \ll L$ & $\theta_s \gg \theta_f \gg \theta_c$
  - soft emissions have the potential to transport energy at large angles
  - they also have a large emission probability $\Longrightarrow$ multiple emissions
- $\Delta P(\omega_0)$: the probability to emit a gluon with energy $\omega > \omega_0$:

$$\Delta P(\omega_0) = \int_{\omega_0}^{\omega_c} \mathrm{d}\omega \, \frac{\mathrm{d}P}{\mathrm{d}\omega} \sim \int_{\omega_0}^{\omega_c} \mathrm{d}\omega \, \frac{\alpha_s}{\omega}\sqrt{\frac{\omega_c}{\omega}} \sim \alpha_s \sqrt{\frac{\omega_c}{\omega_0}}$$

  - integral dominated by its lower limit $\omega = \omega_0$
- When $\omega_0 \sim \alpha_s^2 \omega_c$, this probability becomes of $\mathcal{O}(1)$
  - quasi–deterministic emissions: visible event–by–event
  - a smaller contribution to the energy loss : $\Delta E_{\rm soft} \sim \alpha_s^2 \omega_c$
  - ... but this can be lost at very large angles : $\theta_s \sim \theta_c/\alpha_s^2 \sim 1$ ☺

Multiple emissions

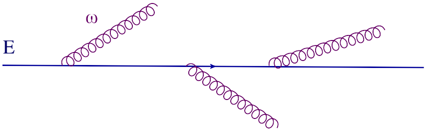
- When $\omega \lesssim \alpha_s^2 \omega_c$: probability for one emission exceeds unity !



$$\Delta P(\omega, L) \simeq \alpha_s \frac{L}{\tau_f(\omega)}$$

Multiple emissions

- Multiple, soft, emissions by the leading particle ('primary gluons')



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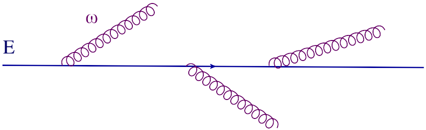
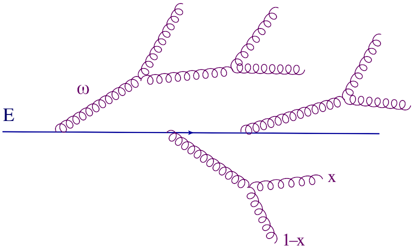


$$\Delta P(\omega, L) \simeq \alpha_s \frac{L}{\tau_f(\omega)}$$

- After being emitted, the soft primary gluons keep on branching



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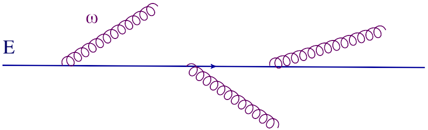

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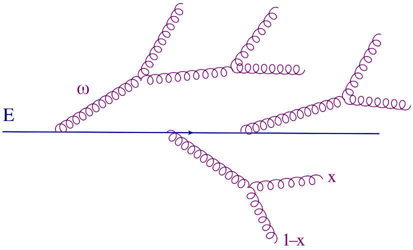
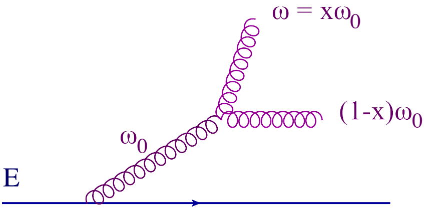


- Their subsequent branchings are quasi–democratic
  - the daughter gluons carry comparable energy fractions: $x \sim 1/2$

Quasi–democratic branchings

- Non–trivial ! Not true for bremsstrahlung in the vacuum !
- Bremsstrahlung in the vacuum : splittings are strongly asymmetric



$$\mathrm{d}P \sim \alpha_s \frac{\mathrm{d}\omega}{\omega} \sim \alpha_s \frac{\mathrm{d}x}{x}$$

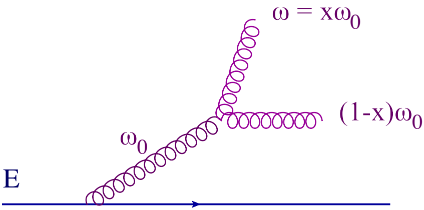
$$\Delta P \sim \alpha_s \int \frac{\mathrm{d}x}{x} \sim \alpha_s \ln \frac{1}{x}$$

- probability of $\mathcal{O}(1)$ when $\alpha_s \ln(1/x) \sim 1 \Longrightarrow$ favors $x \ll 1$
- argument independent of the parent energy ω_0

  ▷ all that matters is the splitting fraction x
- 'soft singularity' $(x \to 0)$ of bremsstrahlung

Quasi–democratic branchings

- In–medium radiation : a consequence of the LPM effect



$$\mathrm{d}P \sim \alpha_s \frac{\mathrm{d}\omega}{\omega}\sqrt{\frac{\omega_c}{\omega}}$$

$$\sim \alpha_s \frac{\mathrm{d}x}{x}\sqrt{\frac{\omega_c}{x\omega_0}}$$

- the rate also depends upon the parent gluon energy ω_0
- probability of $\mathcal{O}(1)$ when $\omega_0 \sim \alpha_s^2\omega_c$ for any value of x
- the phase space favors generic values of x: 'quasi–democratic'

Quasi–democratic branchings

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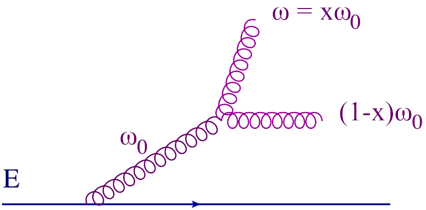

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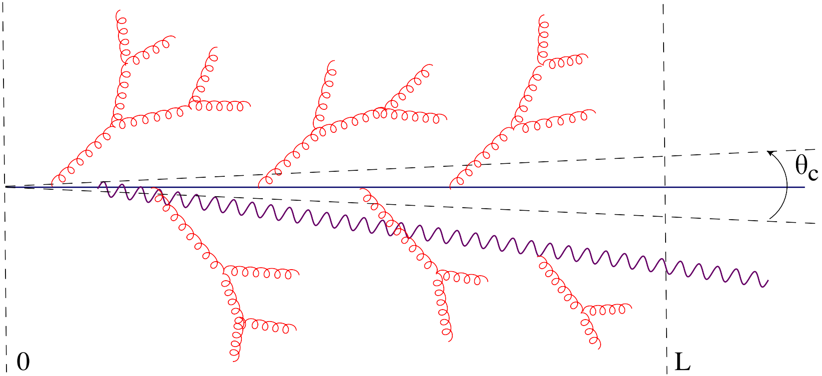
- A similar scenario at strong coupling *(Y. Hatta, E.I., Al Mueller '08)*
- ... but no other known example in a weakly coupled gauge theory

A typical gluon cascade



- The leading particle emits mostly soft gluons ($x \ll 1$)
- The subsequent branchings of these soft gluons are quasi–democratic
- Very efficient in transporting the energy at small x, or large angles