Introduction to pQCD and Jets: lecture 1

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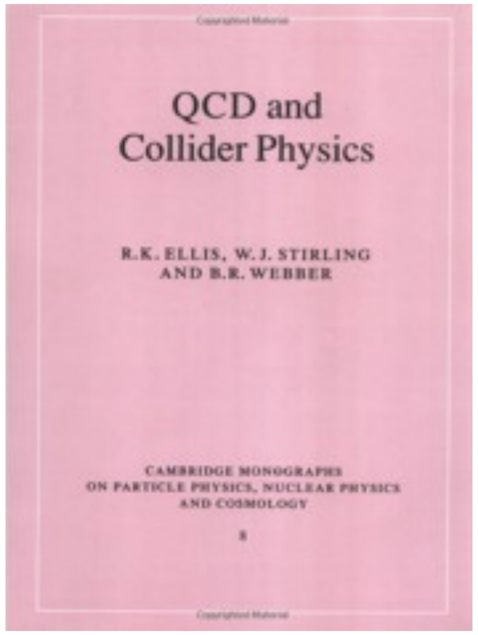
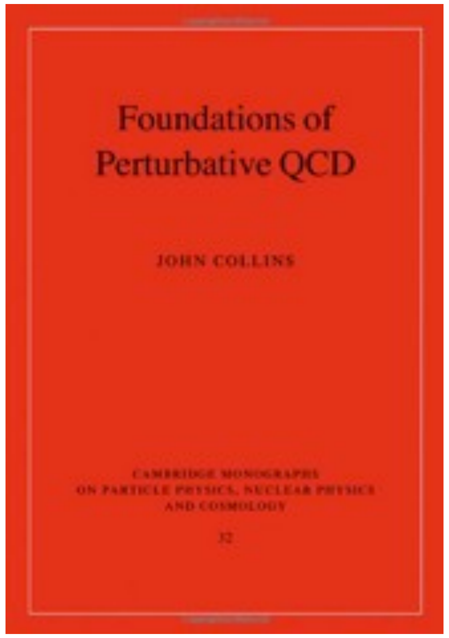
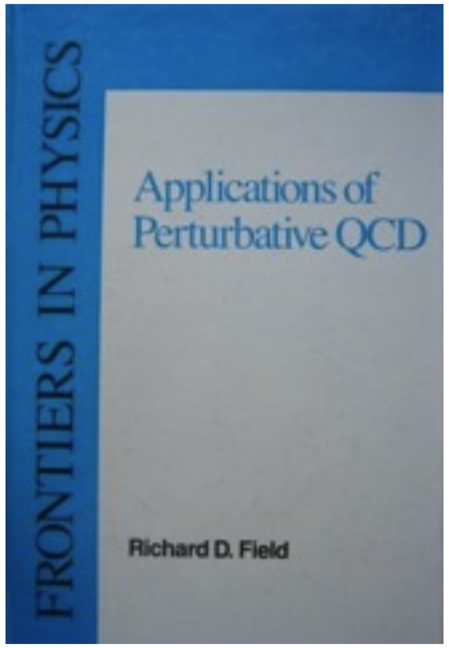
Jet Collaboration Summer School
University of California, Davis
July 19–21, 2014

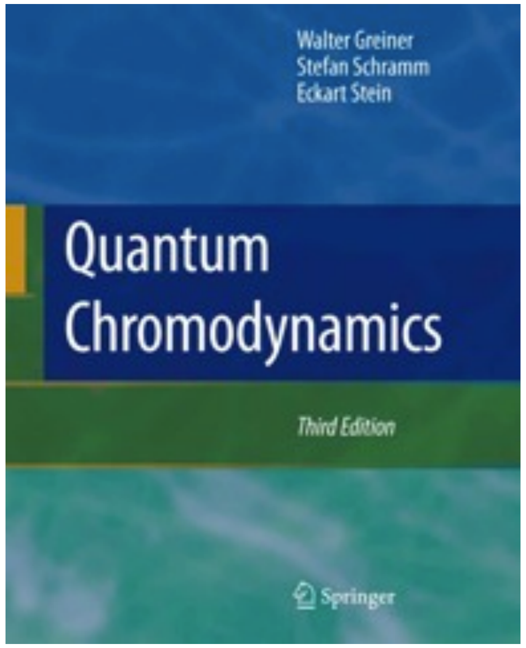
Selected references on QCD

- QCD and Collider Physics: Ellis-Stirling-Webber
- Foundations of Perturbative QCD: J. Collins
- Applications of Perturbative QCD: R. Field
- Quantum Chromodynamics: Greiner-Schramm-Stein





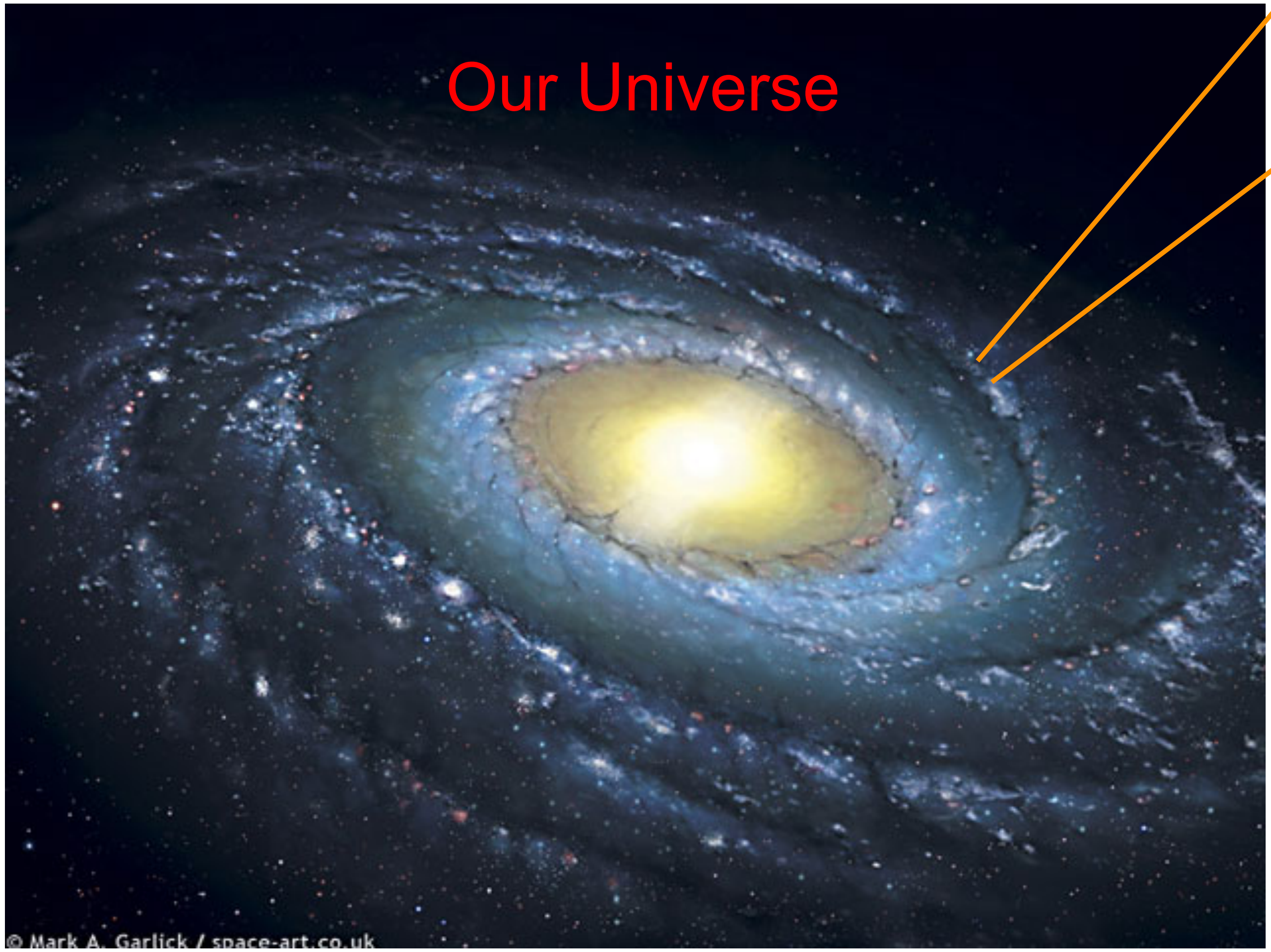




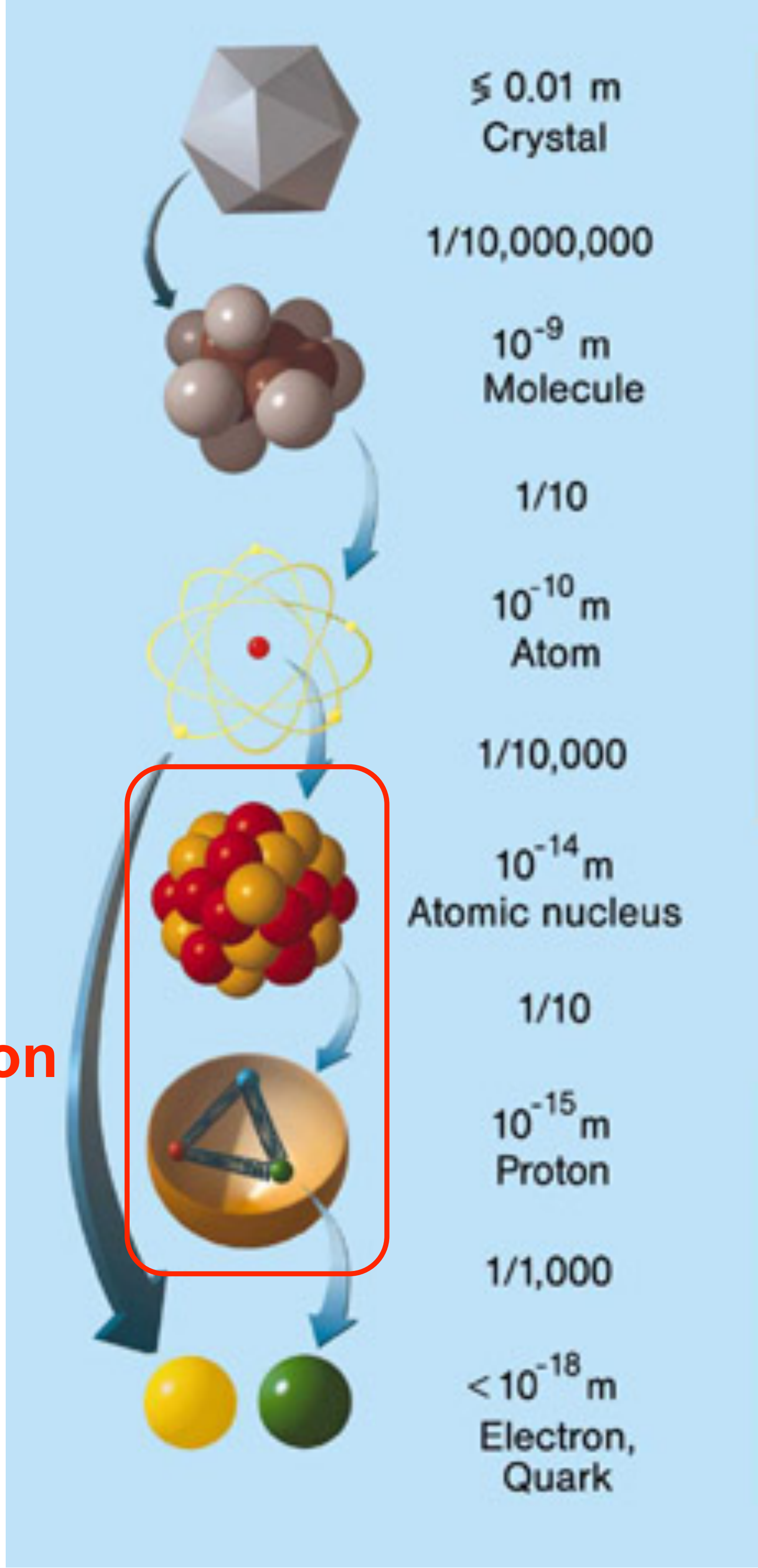
- CTEQ collaboration: http://www.phys.psu.edu/~cteq
- QCD Resource Letter: arXiv:1002.5032 by Kronfeld-Quigg
- Particle Data Group: http://pdg.lbl.gov

We explore the structure of matter (normal and/or QGP)

- The exploration on the structure of matter has a really long history
  - Dalton 1803 (atom)
  - Rutherford 1911 (nucleus)
  - Chadwick 1932 (neutron)
  - Gell-Mann and Zweig 1964 (quark model)
  - Feynman 1969 (parton), ...

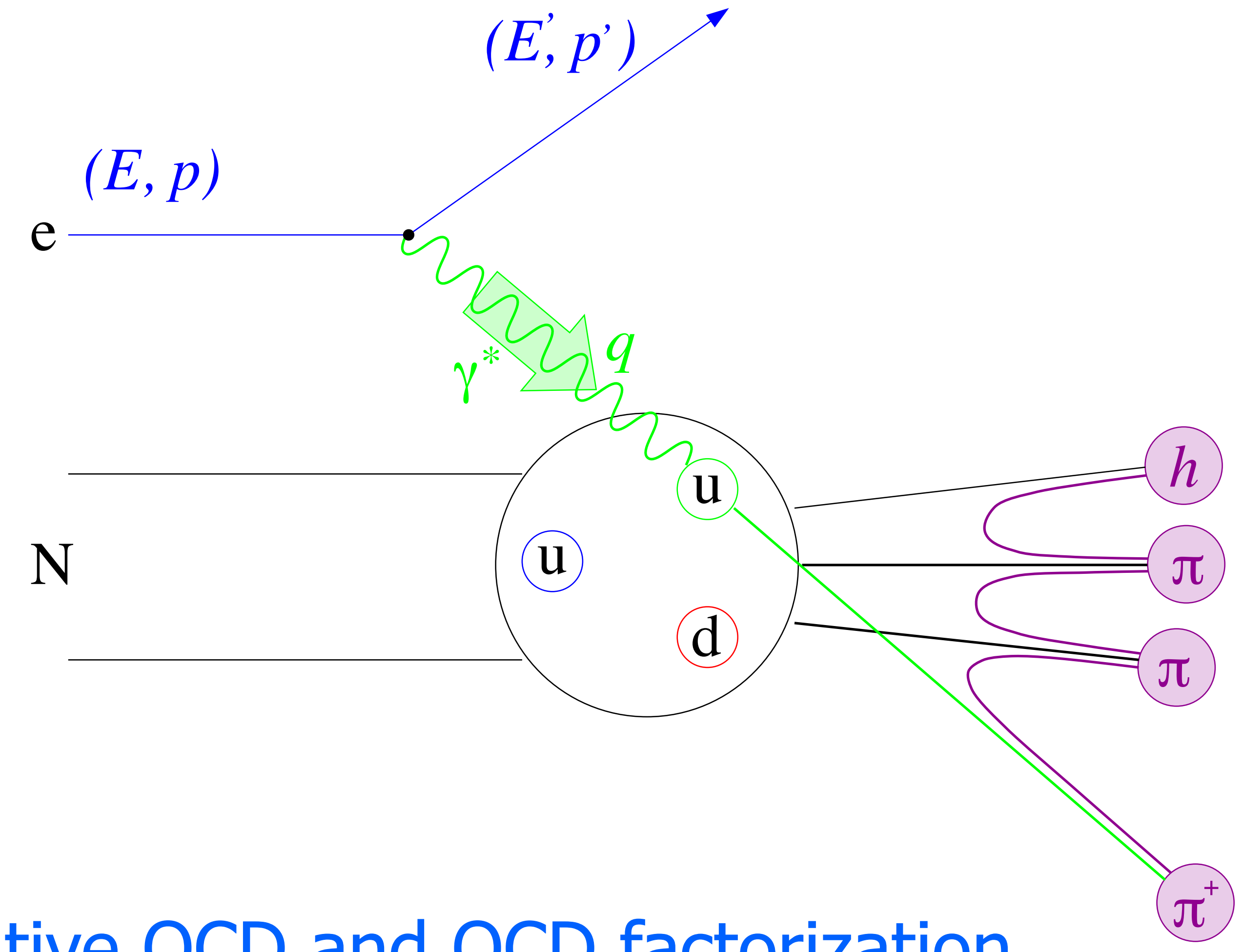


strong interaction



Collider experiments to study hadron structure

- How to study the hadron structure
  - send a high energy probe to collide with the hadron, look for the outcome of the collisions
  - from these out-coming particles, using perturbative QCD, one could trace back to see what's inside the hadron



- Key: perturbative QCD and QCD factorization

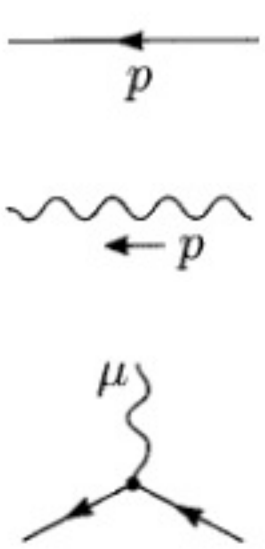
QED: the fundamental theory of electro-magnetic interaction

- QED Lagrangian:

$$\mathcal{L} = \overline{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi + e\,\overline{\Psi}\gamma^\mu \Psi A_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

- Feynman rule: photon has no charge, thus does not self-interact



Dirac propagator: $= \dfrac{i(\not p + m)}{p^2 - m^2 + i\epsilon}$

Photon propagator: $= \dfrac{-ig_{\mu\nu}}{p^2 + i\epsilon}$

QED vertex: $= iQe\gamma^\mu$

($Q = -1$ for an electron)

QCD: the fundamental theory of the strong interaction

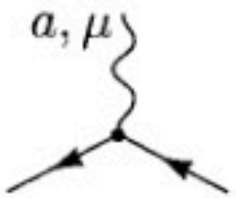
- As the fundamental theory, QCD describes the interaction between quarks and gluons (not hadrons directly)

$$\boxed{\mathcal{L} = \overline{\Psi}_c(i\gamma^\mu\partial_\mu - m)\Psi_c + g\,\overline{\Psi}_c\gamma^\mu T_a\Psi_c G^a_\mu - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a}$$

$$G^a_{\mu\nu} \equiv \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g f_{abc} G^b_\mu G^c_\nu$$

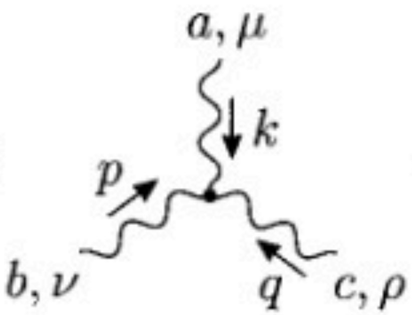
- Feynman rules: gluon carries the color, thus can self-interact

Fermion vertex:



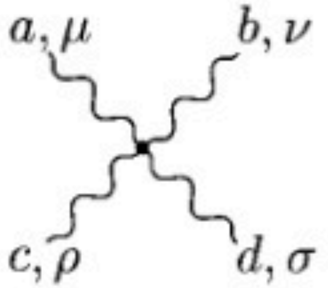
$= ig\gamma^\mu t^a$

3-boson vertex:



$$= g f^{abc}\big[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu\big]$$

4-boson vertex:

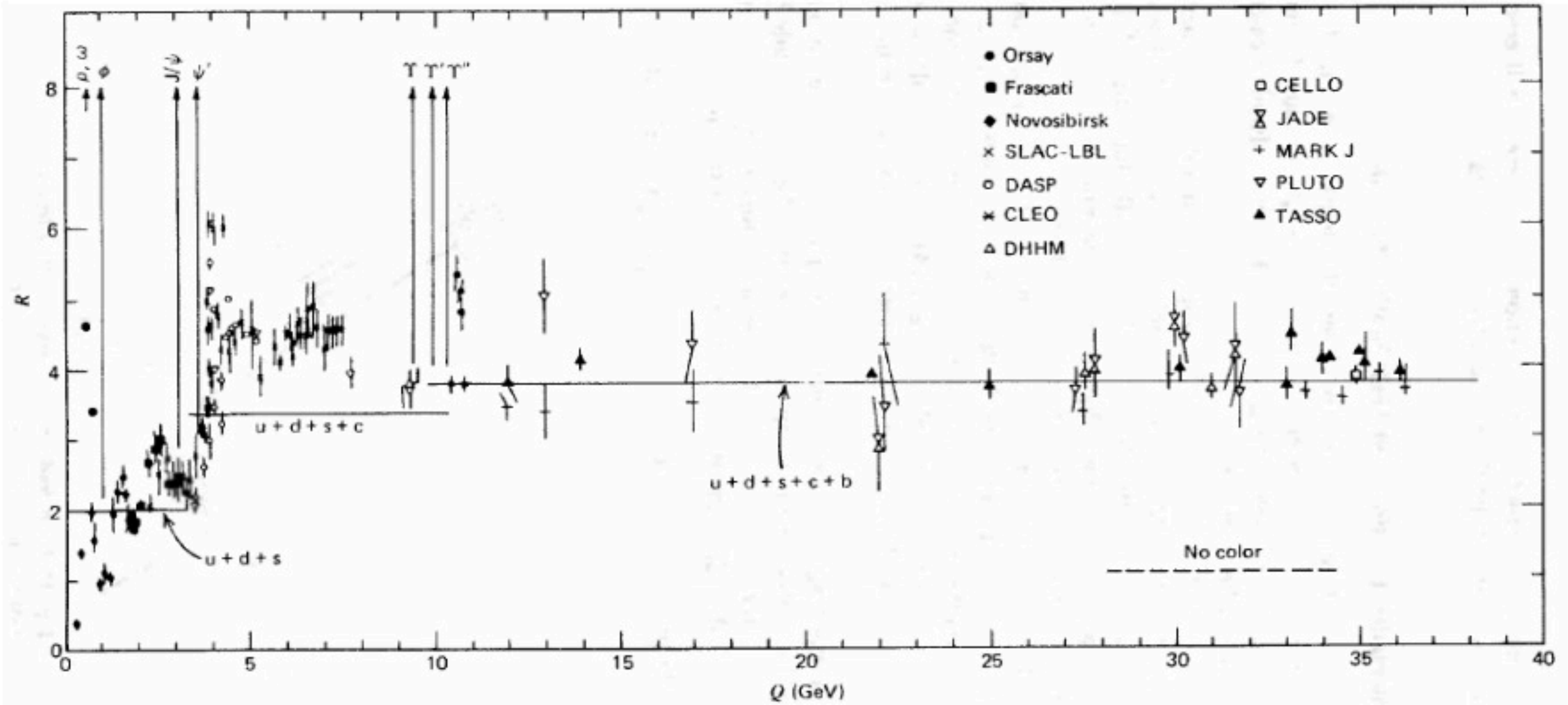


$$= -ig^2\big[f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})\big]$$

Experimental verification about the color

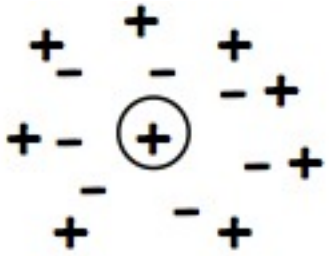
- The color does exist: color of quarks Nc=3 (low energy R=2/3 .vs. 2)

$$R_{e^+e^-} = \frac{\sigma(e^+e^- \to q\bar{q})}{e^+e^- \to \mu^+\mu^-} = N_c \sum_q e_q^2$$

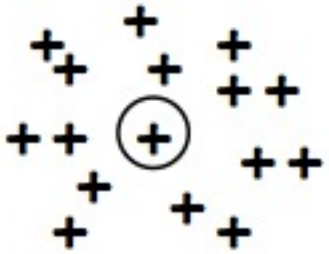


Understanding QCD: the running coupling (Asymptotic freedom)

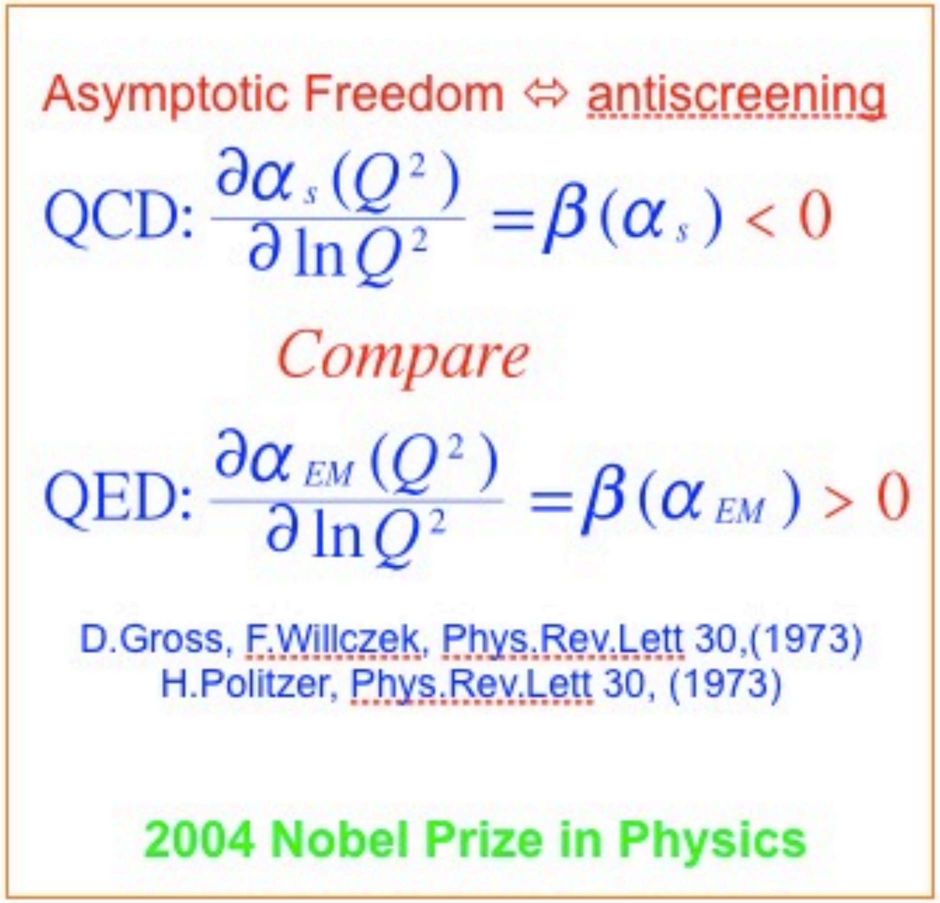
- Rough qualitative picture: due to gluon carrying color charges
  - Value of the strong coupling α_s depends on the distance (i.e., energy)



Screening: $\alpha_{em}(r) \uparrow \text{ as } r \downarrow$



Anti-screening: $\alpha_s(r) \downarrow \text{ as } r \downarrow$



Asymptotic Freedom ⇔ antiscreening

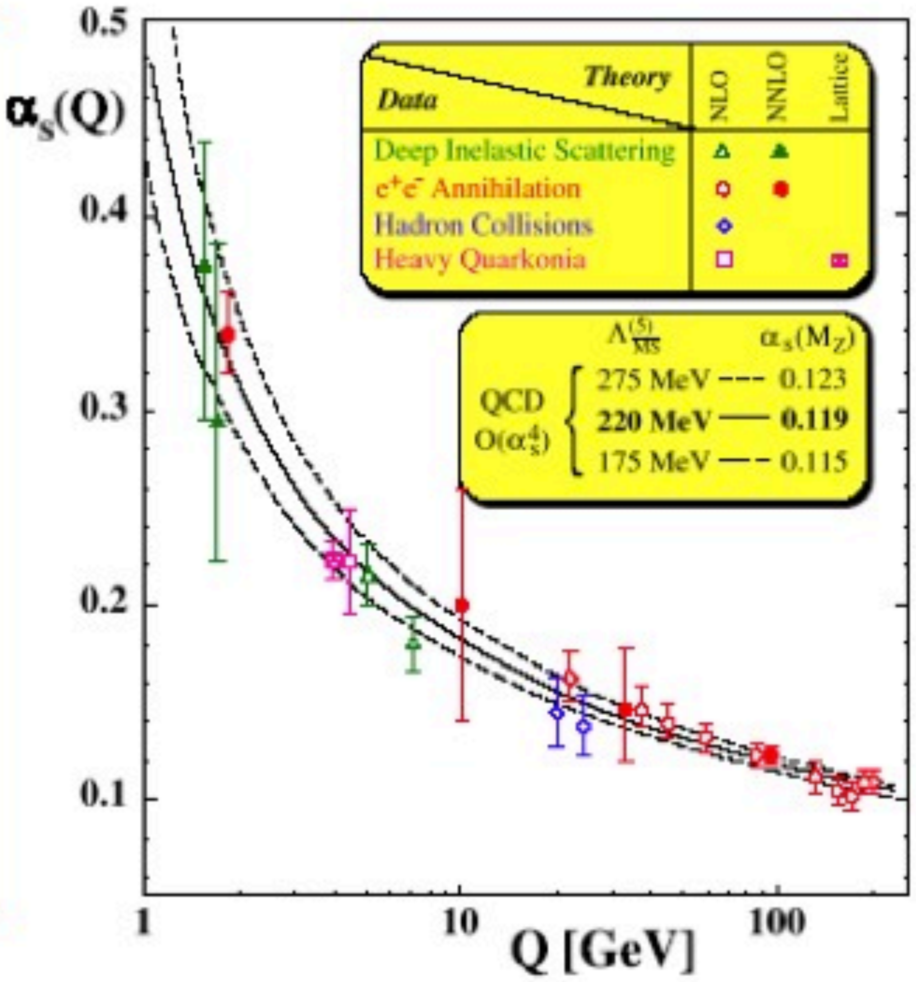
$$\text{QCD: } \frac{\partial \alpha_s(Q^2)}{\partial \ln Q^2} = \beta(\alpha_s) < 0$$

Compare

$$\text{QED: } \frac{\partial \alpha_{EM}(Q^2)}{\partial \ln Q^2} = \beta(\alpha_{EM}) > 0$$

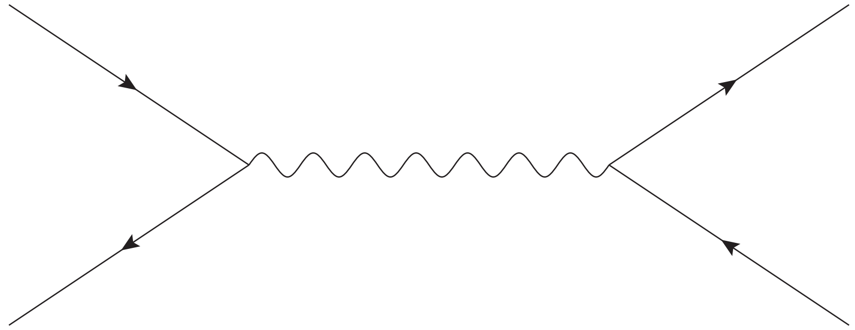
D.Gross, F.Willczek, Phys.Rev.Lett 30,(1973)
H.Politzer, Phys.Rev.Lett 30, (1973)

2004 Nobel Prize in Physics

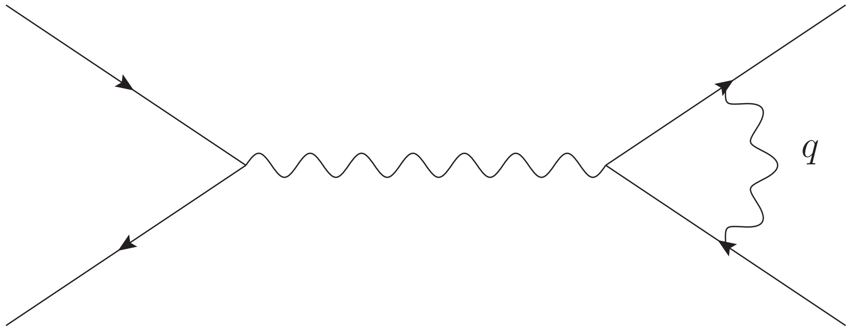


Why does the coupling constant run?

- Leading order calculation is simple: tree diagrams -- always finite



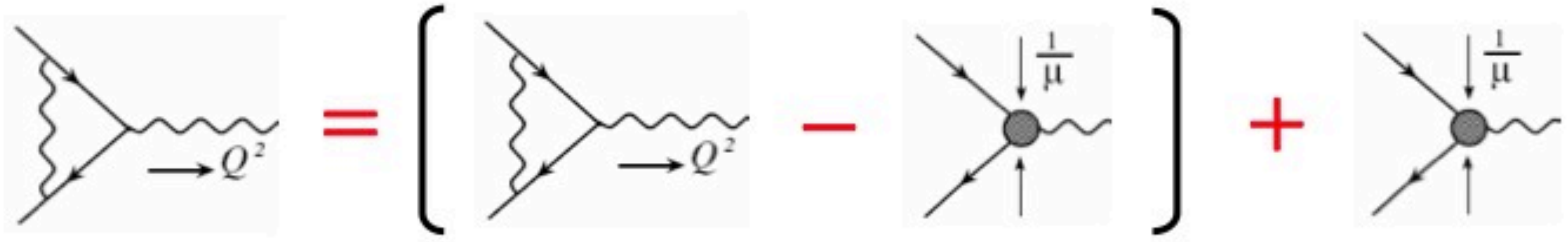
- Study a higher order Feynman diagram: one-loop, the diagram is divergent as q→∞



- Make sense of the result: redefine the coupling constant to be physical

Renormalization (Redefine the coupling constant)

- Renormalization
  - ❖ UV divergence due to “high mass” states
  - ❖ Experiments cannot resolve the details of these states

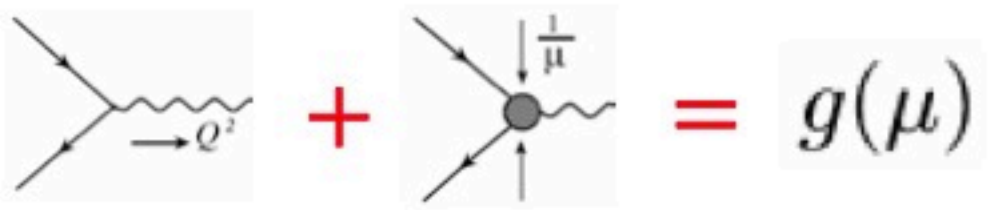


“Low mass” state

“High mass” states

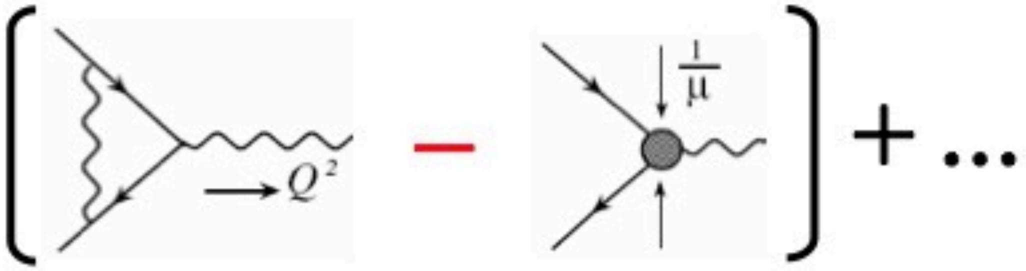
- ❖ combine the “high mass” states with LO

LO:



Renomalized coupling

NLO:



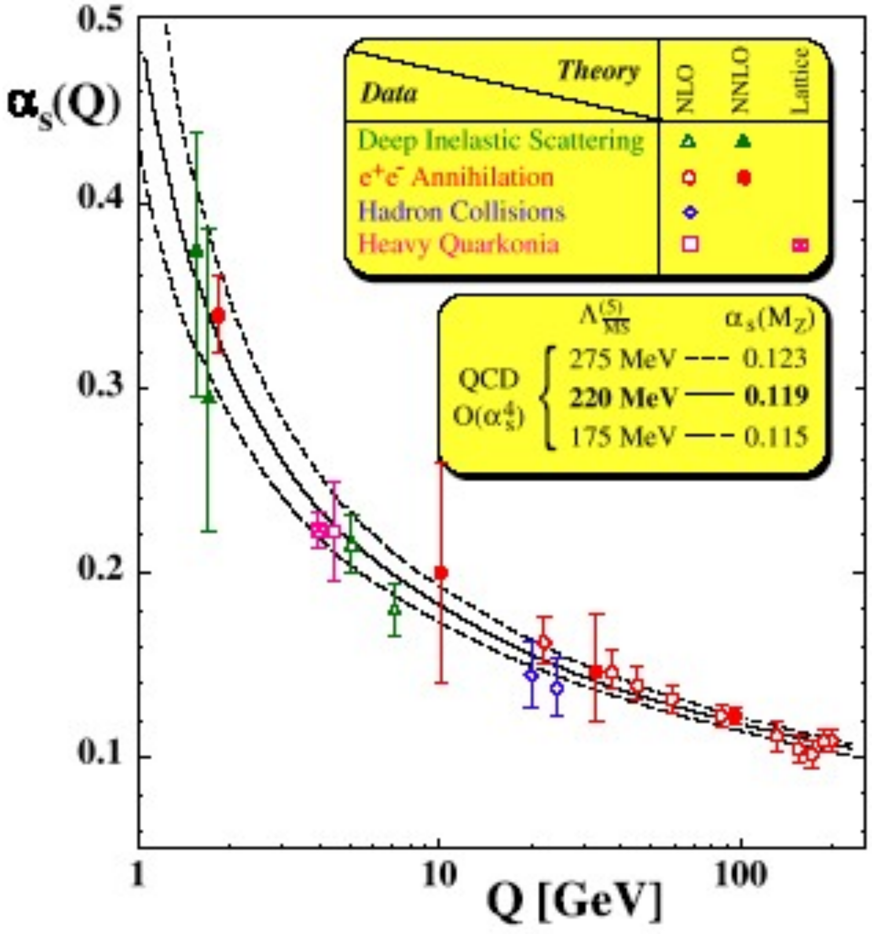
No UV divergence!

Beta-function can be calculated perturbatively

- Since scale μ is an artificial scale we introduced to regulate our calculation, the physical observable (cross section) should be independent of μ, thus study the divergence behavior of the cross section, we could derive how the coupling constant running with the energy scale
- Leading order result

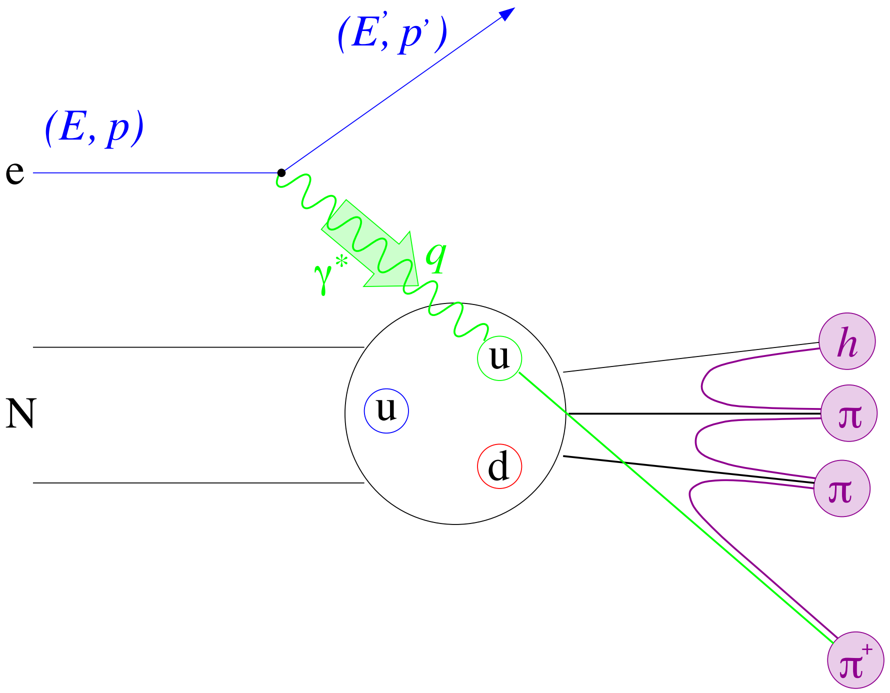
$$\alpha_s(Q^2) = \frac{1}{\beta_0 \log\left(\frac{Q^2}{\Lambda^2_{QCD}}\right)}$$

$$\Lambda_{QCD} \approx 200\ \mathrm{MeV}$$



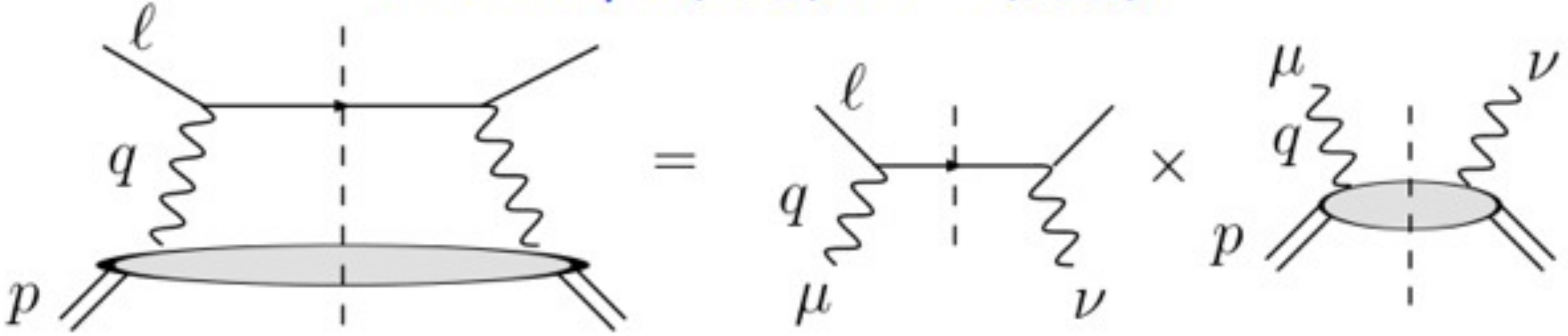
Simple study of Deep Inelastic Scattering: parton model

- DIS has been used a lot in extracting hadron structure



- Leptonic and hadronic tensor

$$d\sigma \propto L_{\mu\nu}(\ell,q)W^{\mu\nu}(p,q)$$



  - Electron is elementary: $L_{\mu\nu}$ can be calculated perturbatively

Structure functions

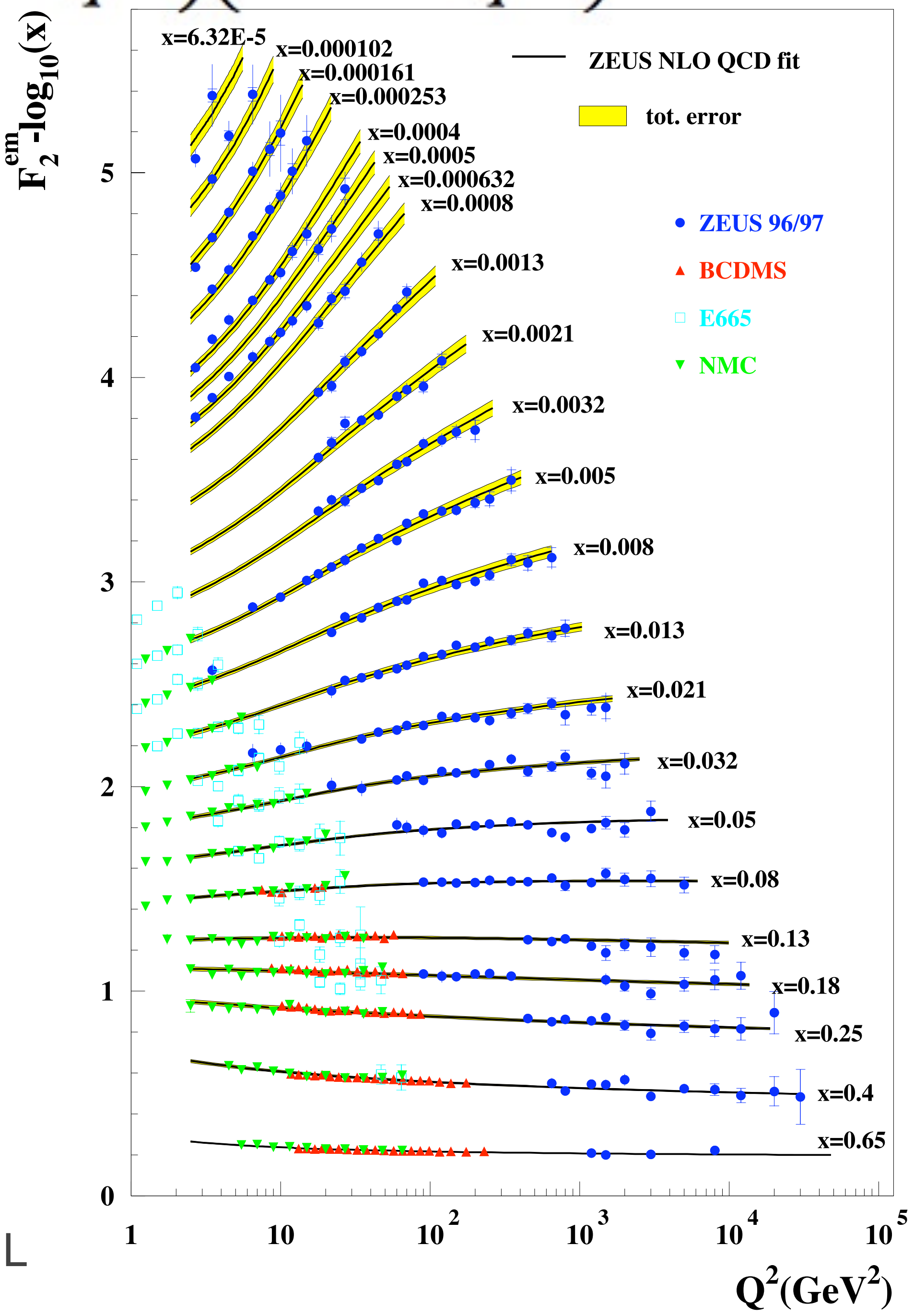
- Hadronic tensor: Lorentz decomposition+parity invariance (for photon case)+time-reversal invariance+gauge invariance

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1\left(x_B,Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right)F_2\left(x_B,Q^2\right)$$

- All the information about hadron structure is contained in the structure functions

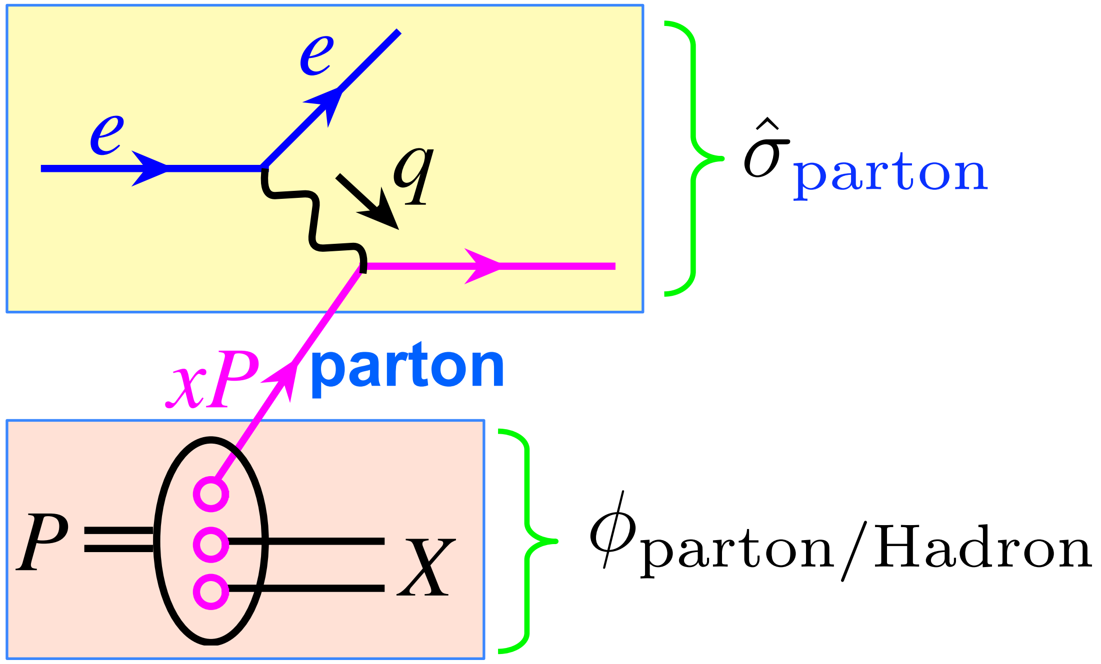
$$L_{\mu\nu} = 2\left(\ell_\mu \ell'_\nu + \ell'_\mu \ell'_\nu - \ell\cdot\ell' g_{\mu\nu}\right)$$

$$\frac{d\sigma}{dx_B dQ^2} = \frac{4\pi\alpha^2}{x_B Q^4}\left\{\left(1-y-x_B^2 y^2\frac{M^2}{Q^2}\right)F_2(x_B,Q^2) + y^2 x_B F_1(x_B,Q^2)\right\}$$



Parton model picture

- Photon interact with parton: deep inelastic scattering e+p→e+X



Parton Distribution Functions (PDFs): probability density for finding a parton in a hadron with longitudinal momentum fraction *x*

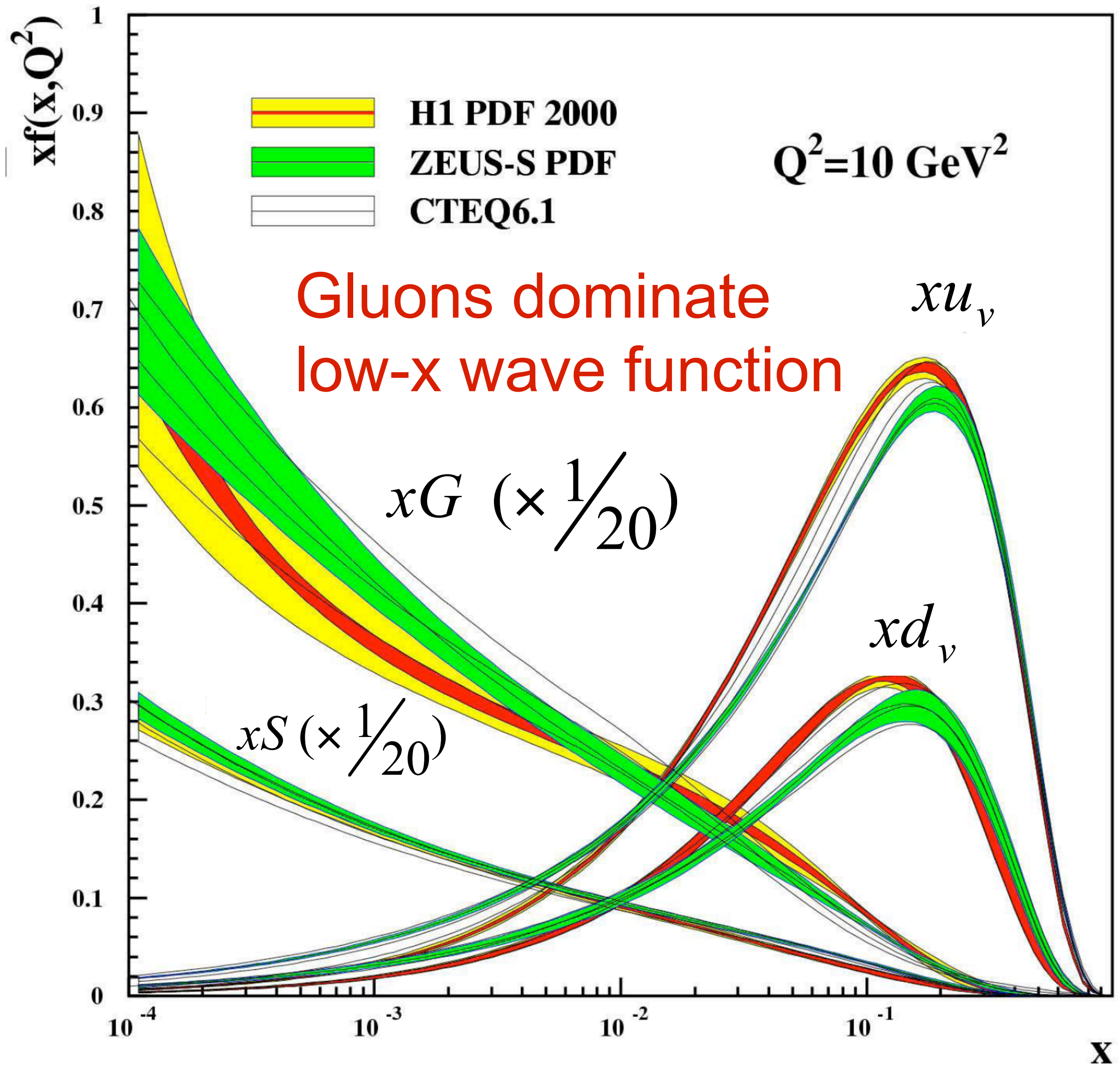
$$\sigma_{\text{Hadron}}(Q) = \phi_{\text{parton/Hadron}}(\Lambda_{QCD}) \otimes \hat{\sigma}_{\text{parton}}(Q)$$

Universal (measured) **calculable**

- Hadron structure: encoded in PDFs
- QCD dynamics at short-distance: partonic cross section, perturbatively calculable

Parton distribution functions

- By measuring the structure functions (cross sections) in DIS, one could trace back to find the parton distribution functions inside the proton by comparing the data with the theoretical formalism

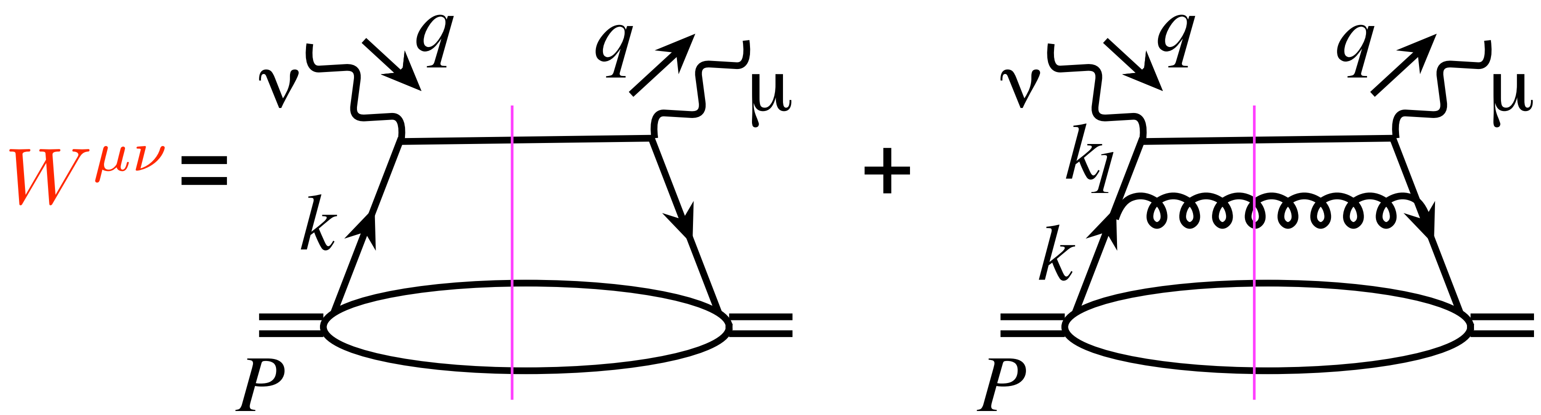


Gluons dominate low-x wave function

What about higher order?

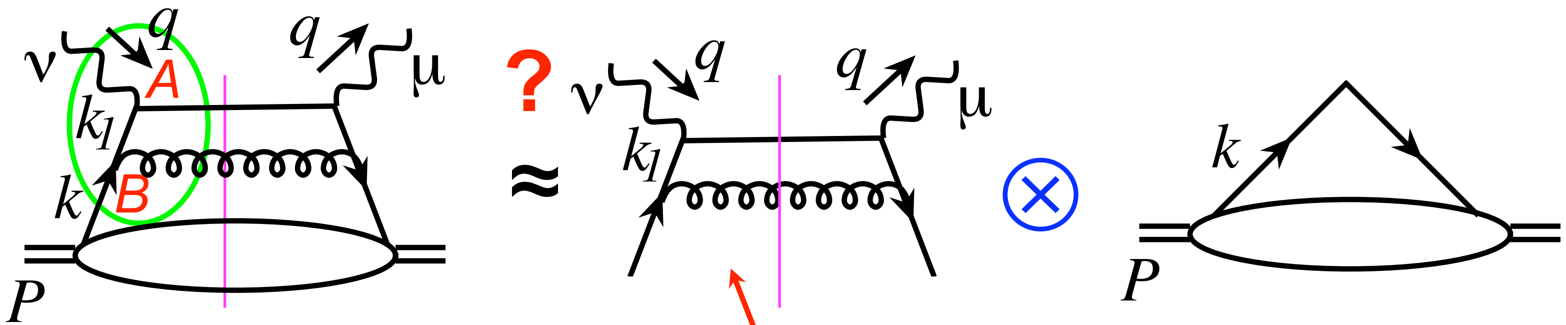
- pQCD calculations: understand and make sense of all kinds of divergences
  - Ultraviolet (UV) divergence $k \to \infty$: renormalization (redefine coupling constant)
  - Collinear divergence $k // P$: redefine the PDFs and FFs
  - Soft divergence $k \to 0$: usually cancel between real and virtual diagram for collinear PDFs/FFs; do not cancel for kt-dependent PDFs/FFs, leads to new evolution equations

- If going beyond the leading order of the DIS, we face another divergence



QCD dynamics beyond tree level

- Going beyond leading order calculation



Collinear divergence!!! (from $k_1^2 \sim 0$)

$$\Rightarrow \int d^4k_1 \frac{i}{k_1^2+i\epsilon}\,\frac{-i}{k_1^2-i\epsilon} \Rightarrow \infty$$

$$k_1^2 = (k+k_g)^2 = 2EE_g(1-\cos\theta)$$

- ❖ $k_1^2 \sim 0$ intermediate quark is on-shell

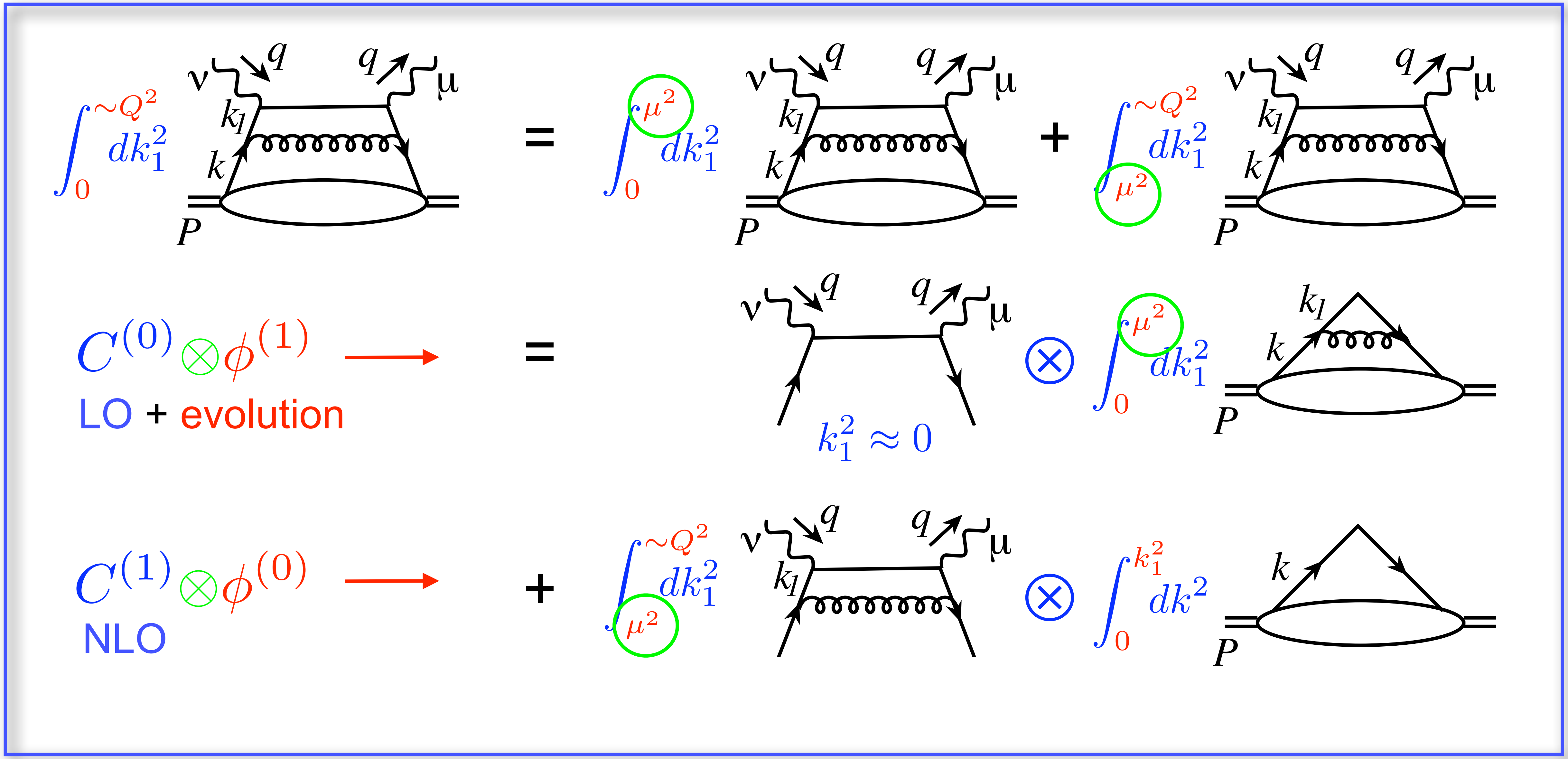
  $t_{AB} \rightarrow \infty$

- ❖ gluon radiation takes place long before the photon-quark interaction
  $\Longrightarrow$ a part of PDF

Partonic diagram has both long- and short-distance physics

QCD factorization: beyond parton model

- Systematic remove all the long-distance physics into PDFs



$\int_0^{\sim Q^2} dk_1^2 \; = \; \int_0^{\mu^2} dk_1^2 \; + \; \int_{\mu^2}^{\sim Q^2} dk_1^2$

$C^{(0)} \otimes \phi^{(1)} \longrightarrow \; = \; k_1^2 \approx 0 \; \otimes \int_0^{\mu^2} dk_1^2$

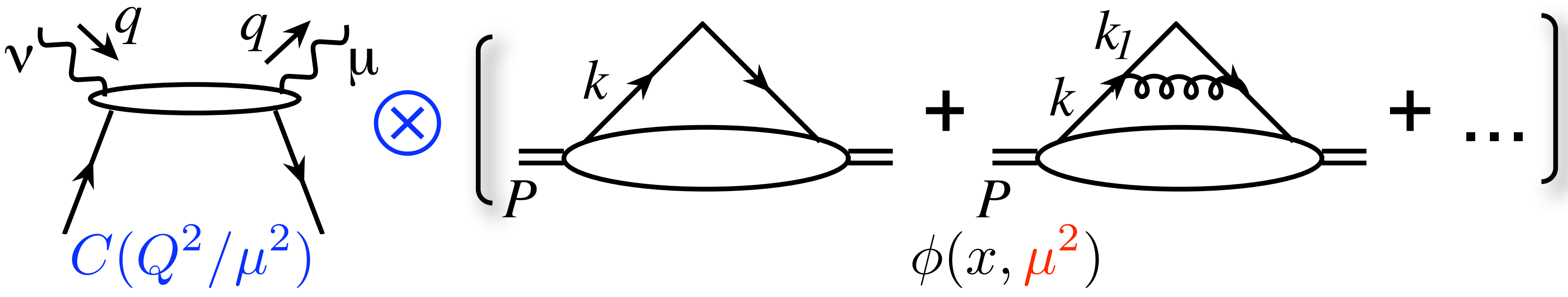
LO + evolution

$C^{(1)} \otimes \phi^{(0)} \longrightarrow \; + \int_{\mu^2}^{\sim Q^2} dk_1^2 \; \otimes \int_0^{k_1^2} dk^2$

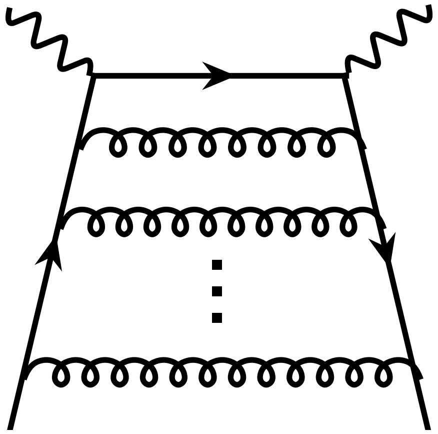
NLO

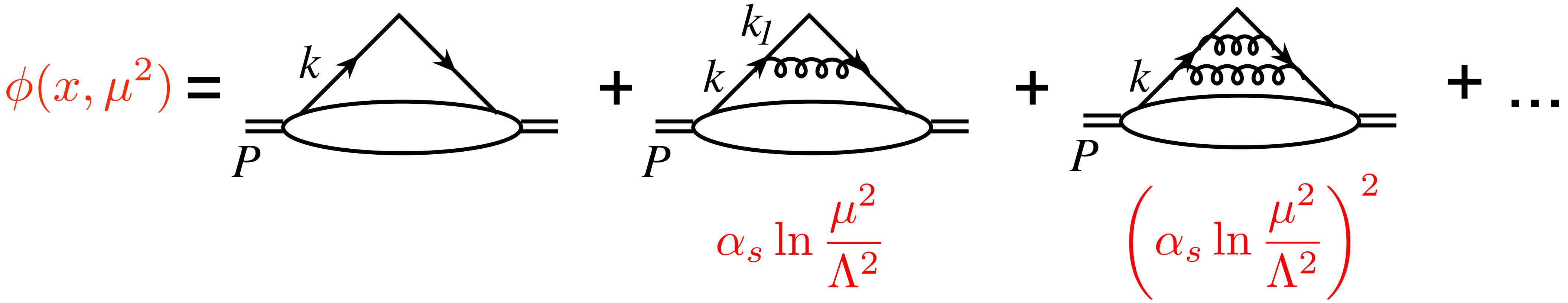
Scale-dependence of PDFs

- Logarithmic contributions into parton distributions



- Going to even higher orders: QCD resummation of single logs

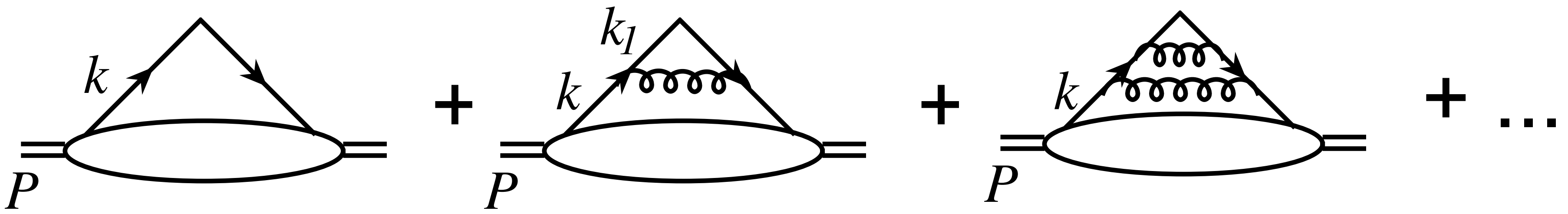




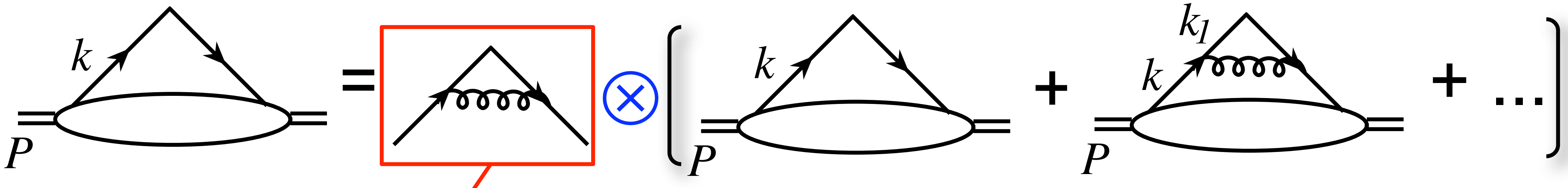
DGLAP evolution = resummation of single logs

- Evolution = Resum all the gluon radiation

$\phi(x,\mu^2) =$



$\phi(x,\mu^2)$ -



Evolution kernel
splitting function

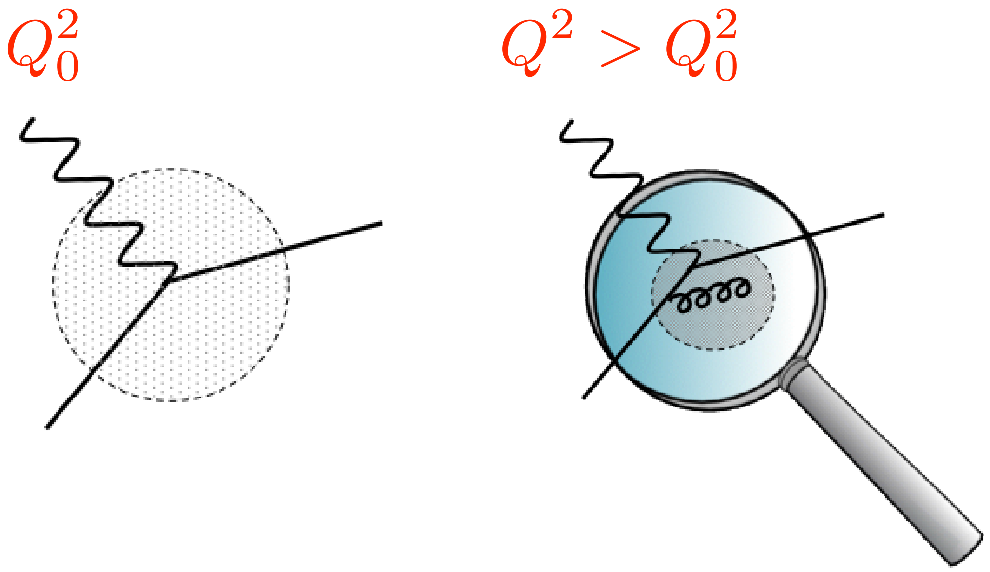
➡ **DGLAP Equation**

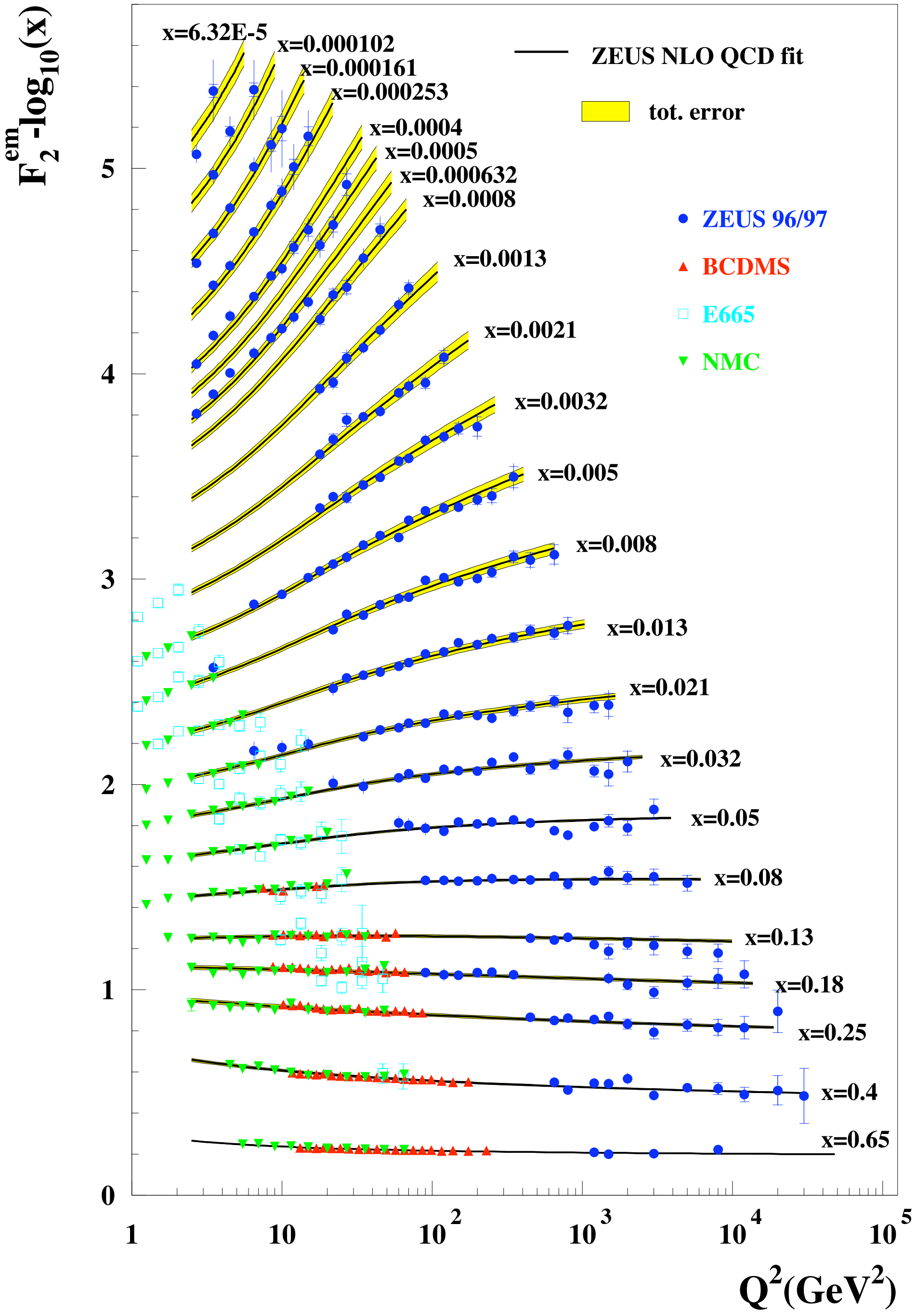
$$\frac{\partial}{\partial \ln \mu^2}\phi_i(x,\mu^2) = \sum_j \boxed{P_{ij}(\frac{x}{x'})} \otimes \phi_j(x',\mu^2)$$

- By solving the evolution equation, one resums all the single logarithms of $\left(\alpha_s \ln\frac{\mu^2}{\Lambda^2}\right)^n$

Parton distribution also depends on the scale of the probe

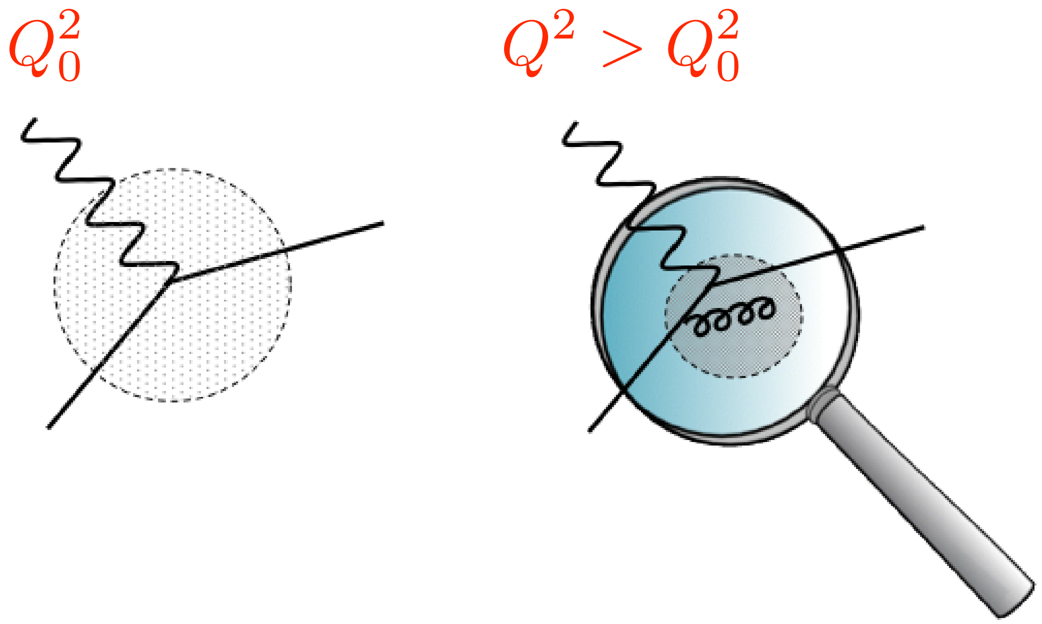
- Increase the energy scale, one sees parton picture differently



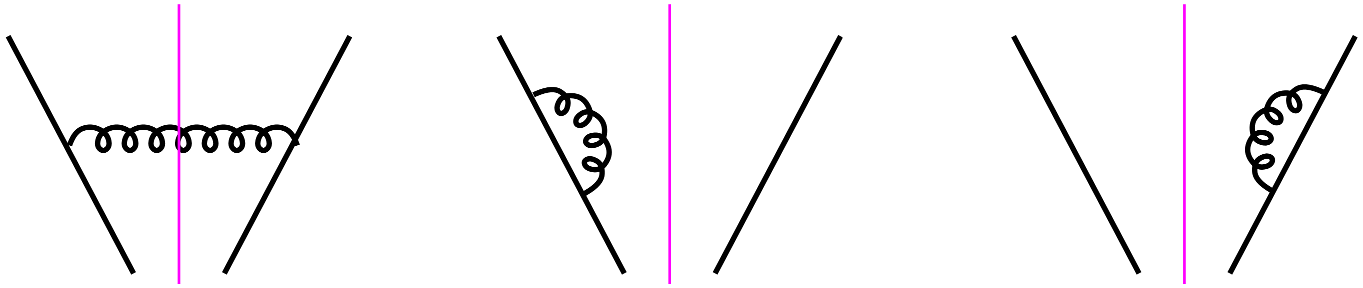


Evolutions of parton distribution functions

- Perturbative change:



- Feynman diagrams for unpolarized PDF (non-singlet case):



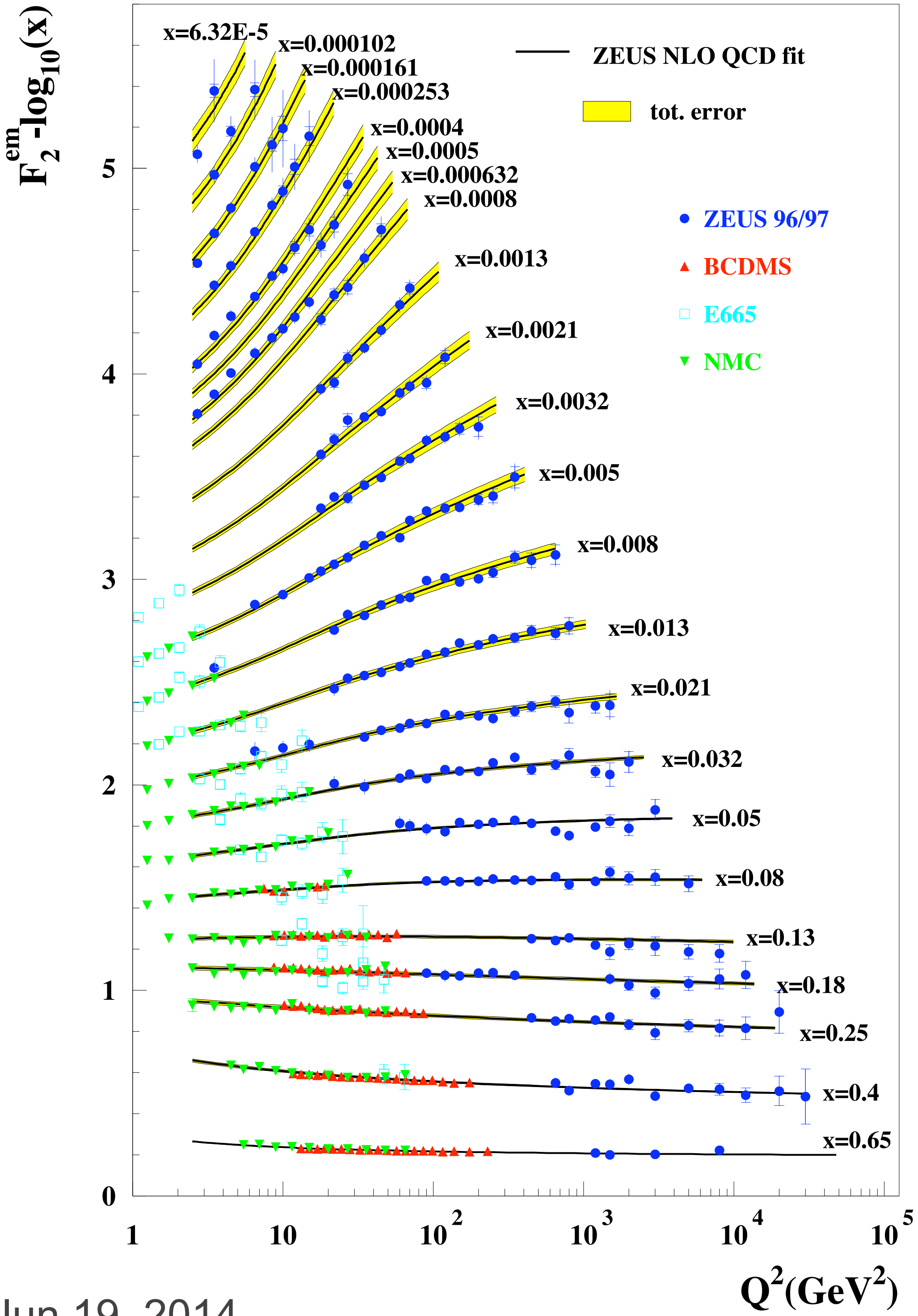
$$\frac{q(x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[P_{qq}(z)\, q(\xi,\mu_F)\right]$$

$$P_{qq}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$

Success of QCD factorization

- Universality of PDFs: mapped in one process (say DIS), used in other process (**p+p→jet+X**)

DIS



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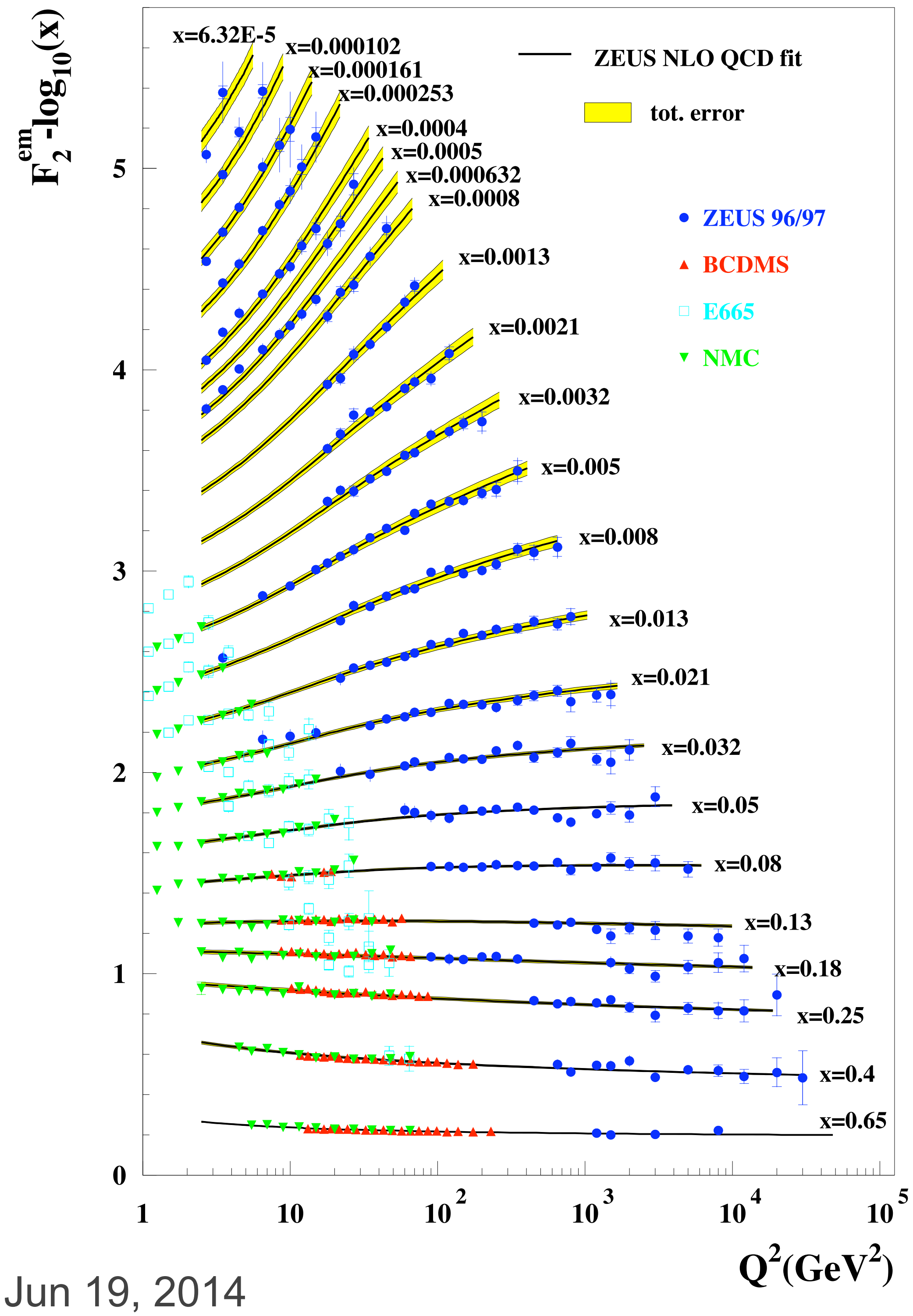

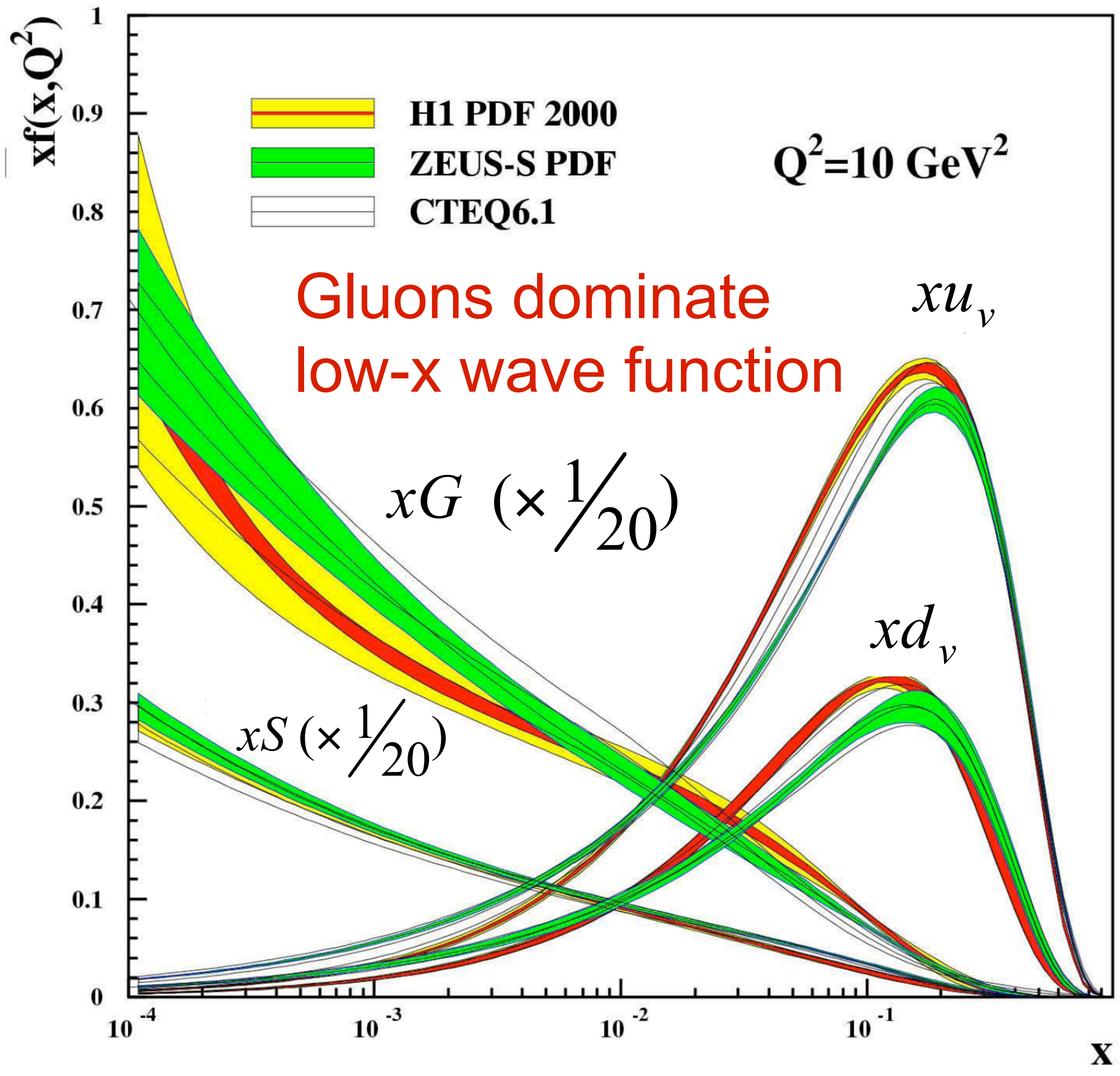

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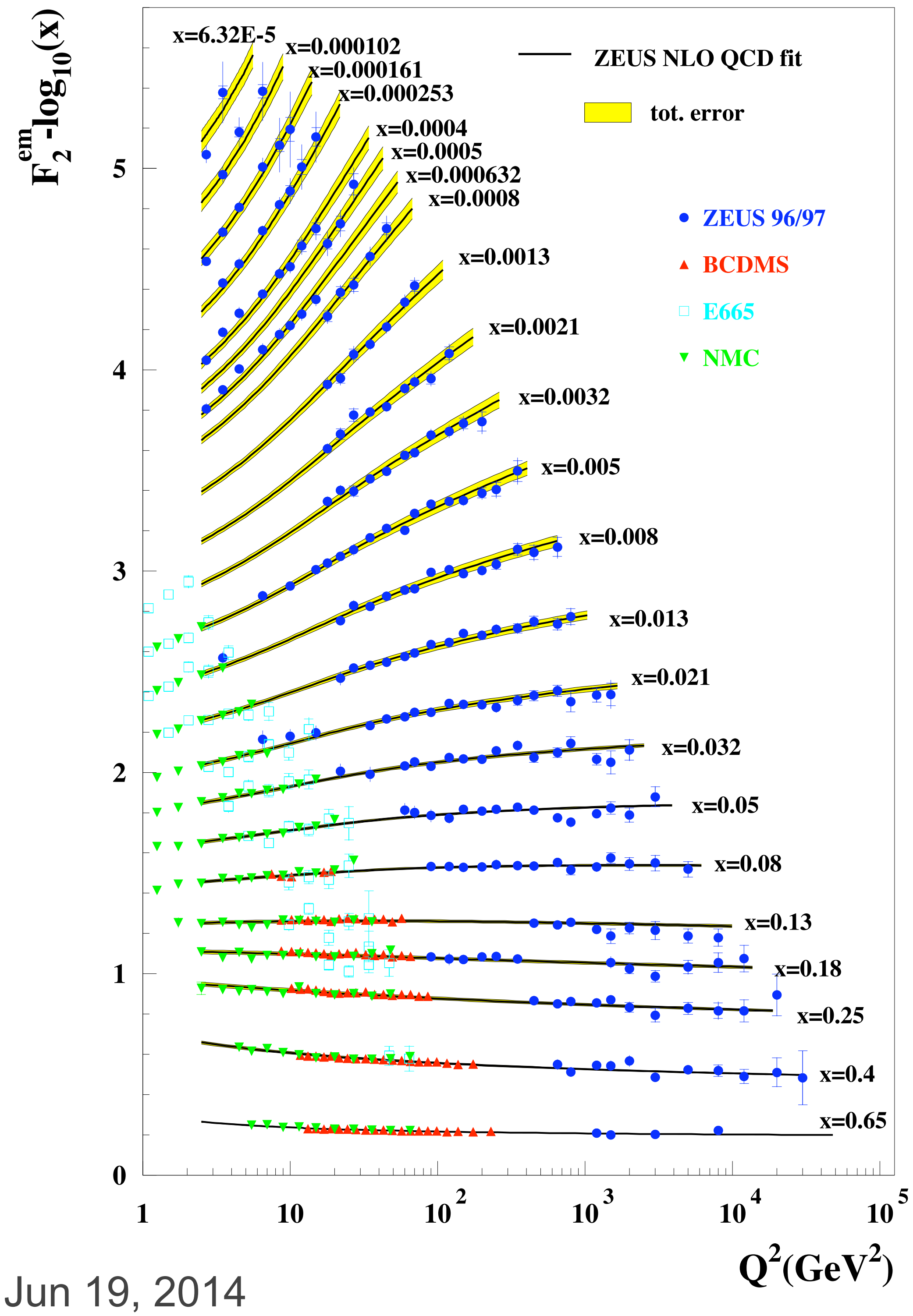
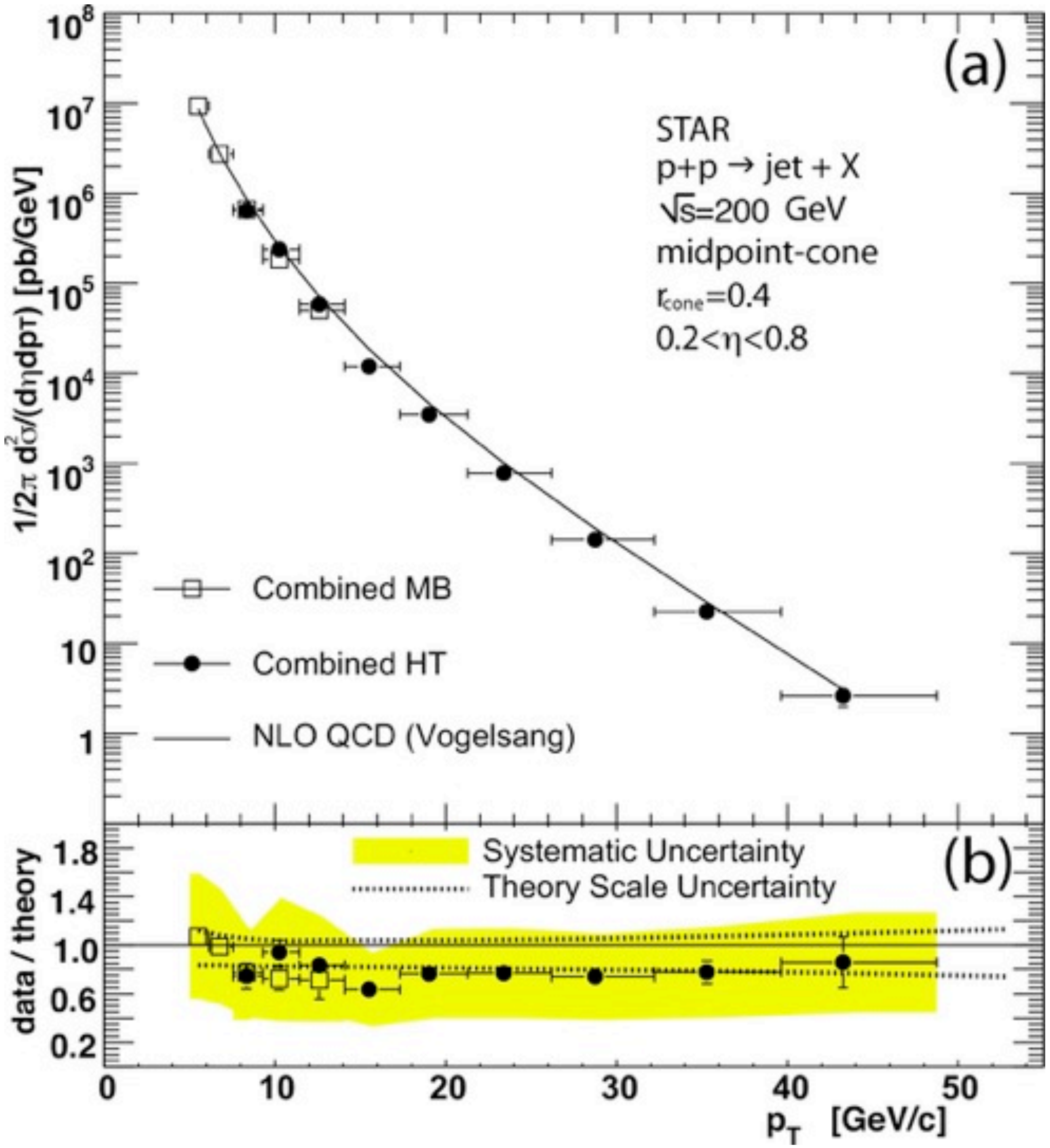


RHIC 200 GeV



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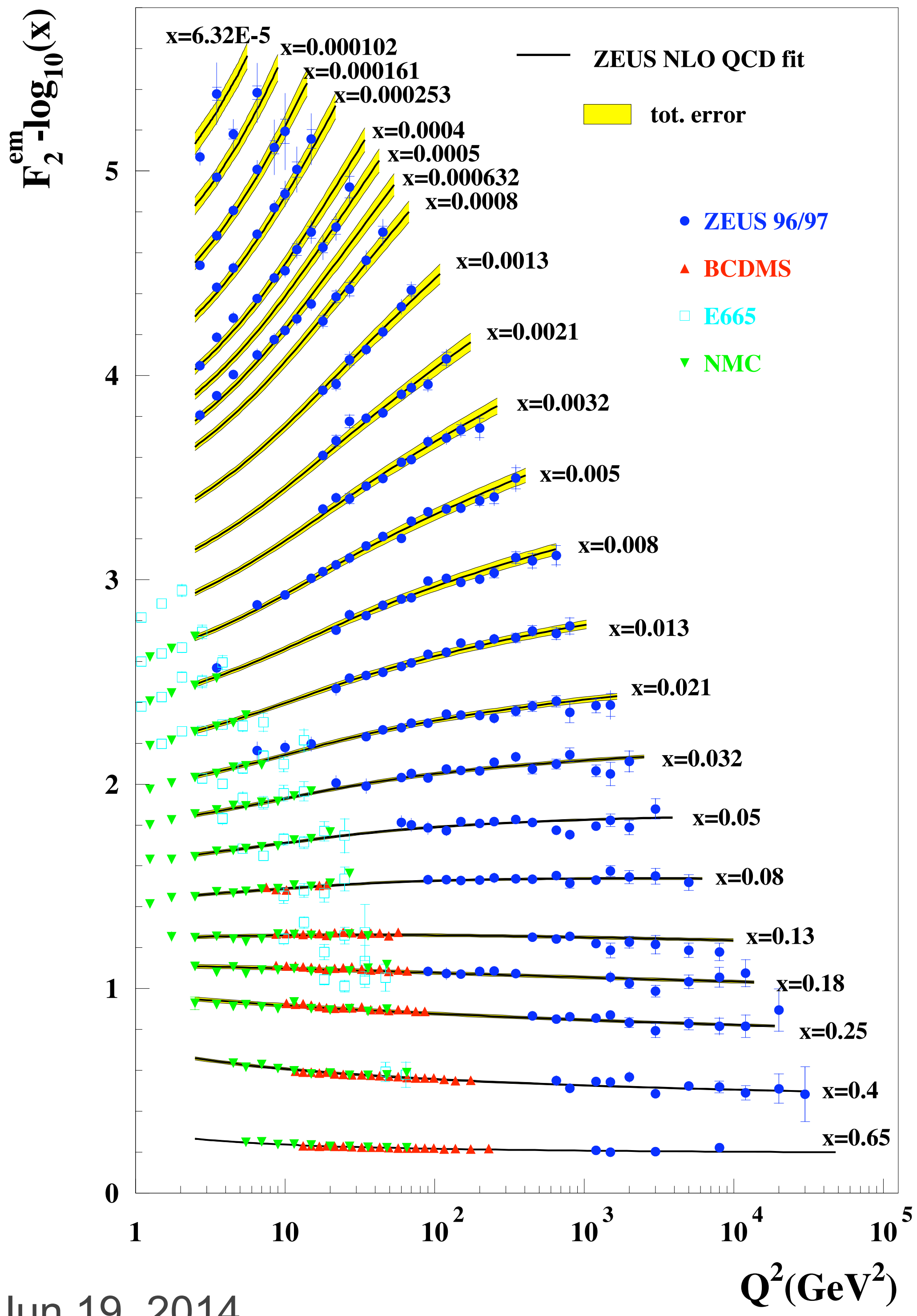
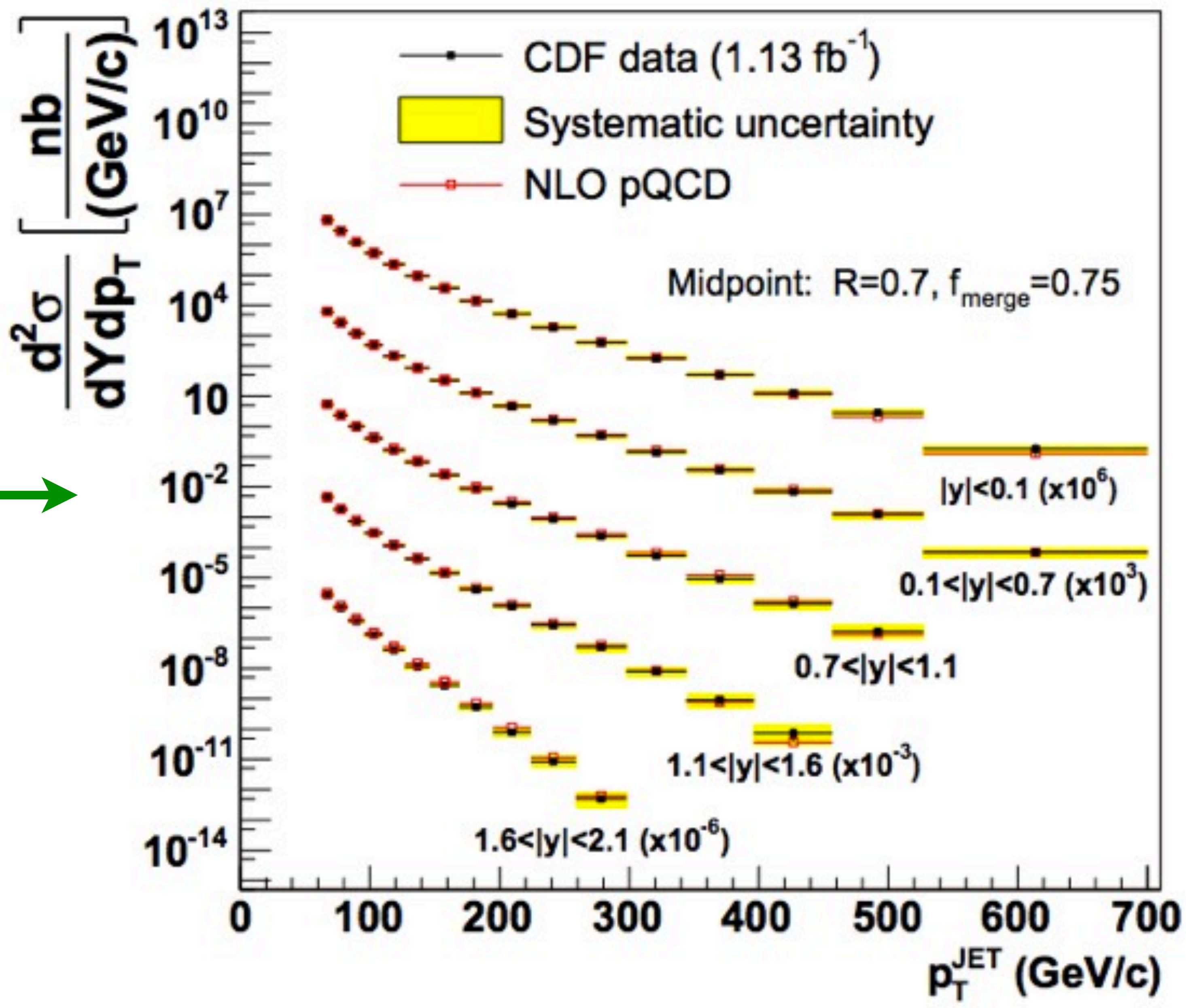


Tevatron 1.96 TeV



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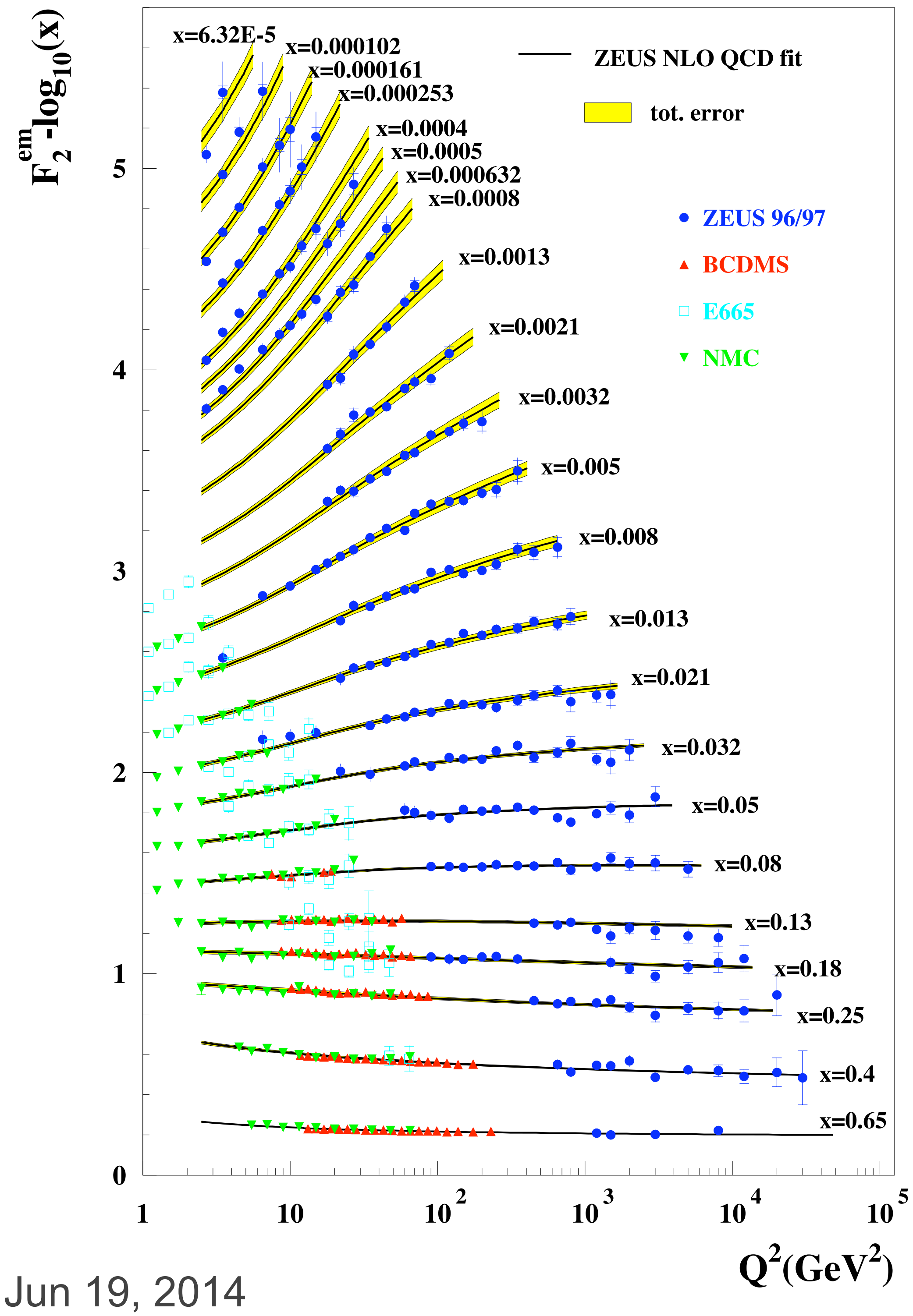
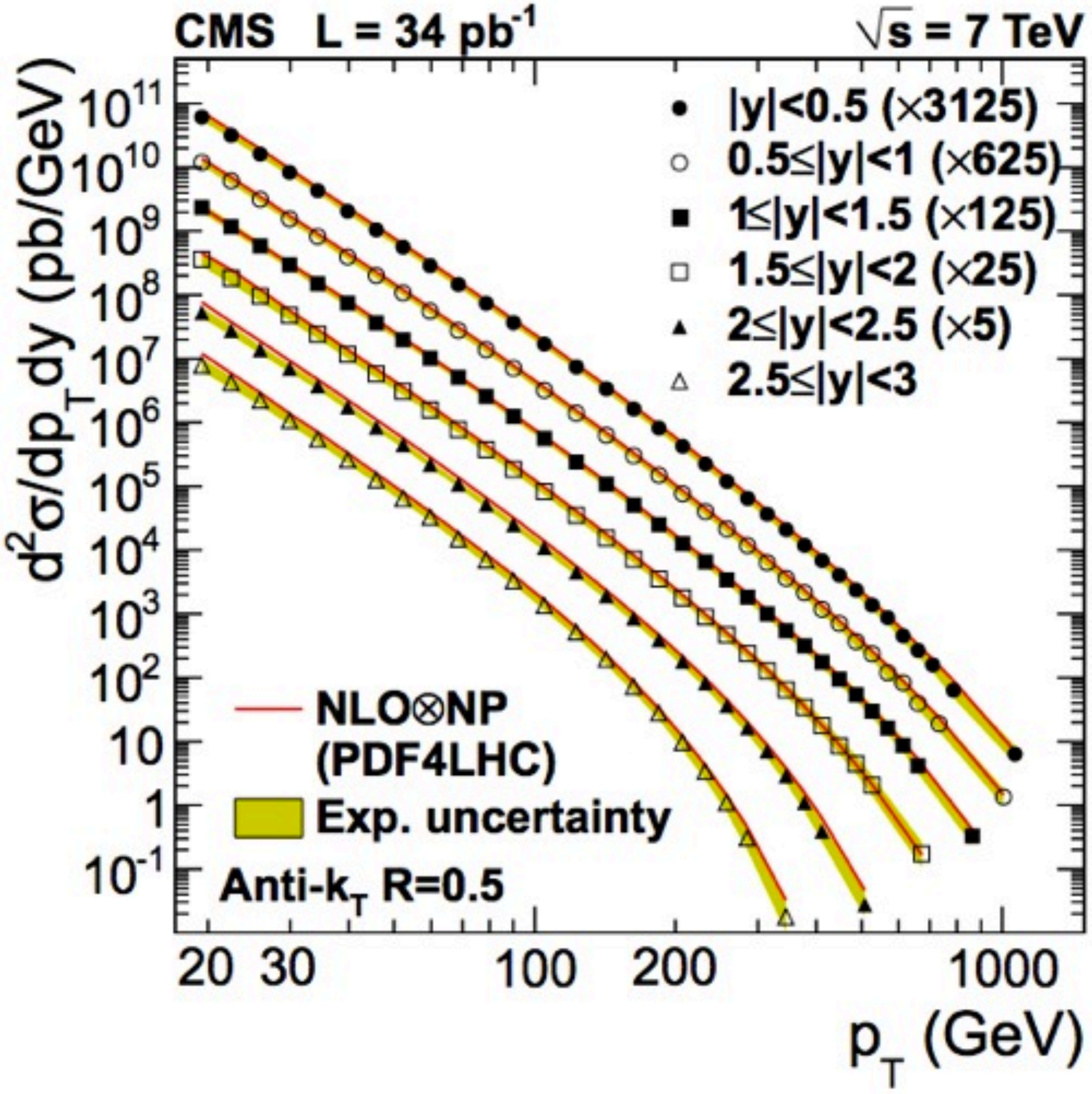
- Universality of PDFs: mapped in one process (say DIS), used in other process (**p+p→jet+X**)

DIS

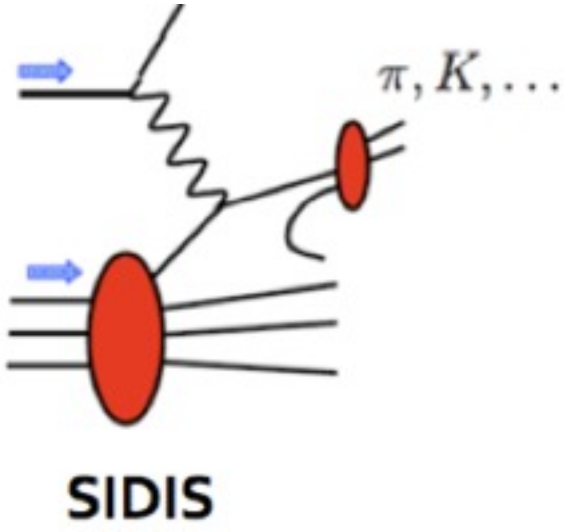


LHC 7 TeV



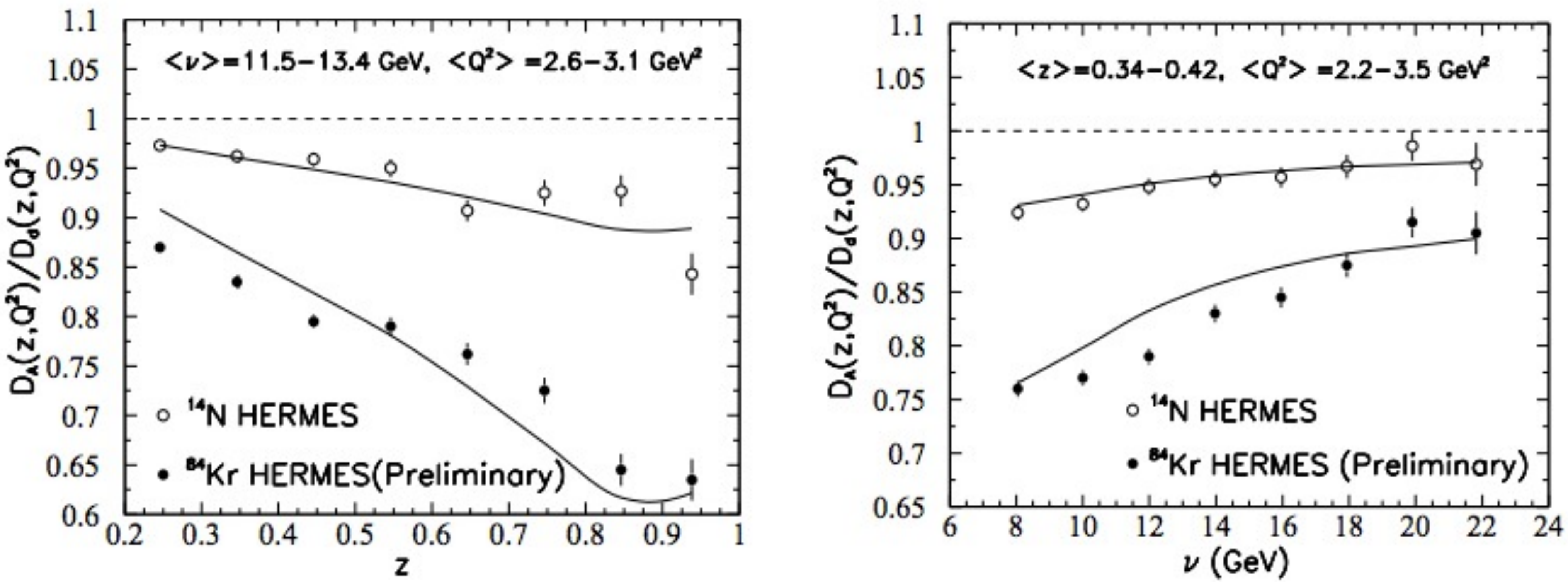
An explicit example: SIDIS (detailed note for your convenience)

- Semi-inclusive deep inelastic scattering



- Connection to high energy nuclear physics (heavy ion physics)

Higher-twist approach to energy loss, Wang-Guo, PRL,2000
Energy loss at HERMES, Wang-Wang, PRL, 2002



Summary

- pQCD provides a way to extract information on hadron structure
    - Asymptotic freedom: allow one to calculate partonic cross sections
    - Parton distribution functions
    - Renormalization scale and factorization scale

Summary

- pQCD provides a way to extract information on hadron structure
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  - Parton distribution functions
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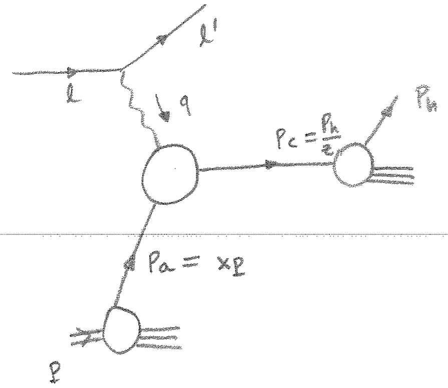
Thank you



$$S = (P+l)^2 \approx 2P\cdot l$$

$$Q^2 = -q^2$$

$$x_B = \frac{Q^2}{2P\cdot q} \qquad z_h = \frac{P\cdot P_h}{P\cdot q} \qquad y = \frac{P\cdot q}{P\cdot l} = \frac{Q^2}{x_B S}$$

define $\hat{x} = \frac{x_B}{x}$ $\qquad \hat{z} = \frac{z_h}{z}$

work in the so-called hadron frame $\quad \bar{n}^\mu = [1^+, 0^-, 0_\perp] \quad n^\mu = [0^+, 1^-, 0_\perp]$

$$p^\mu = p^+ \bar{n}^\mu$$

$$q^\mu = -x_B p^+ \bar{n}^\mu + \frac{Q^2}{2x_B p^+} n^\mu$$

from $z_h = \frac{P\cdot P_h}{P\cdot q} = \frac{P^+ P_h^-}{Q^2/(2x_B)} \Rightarrow P_h^- = z_h \frac{Q^2}{2x_B p^+}$

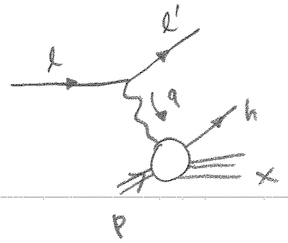
$$P_h^2 = 2P_h^+ P_h^- - \vec{P}_{h\perp}^2 \Rightarrow P_h^+ = \frac{\vec{P}_{h\perp}^2}{2P_h^-} = \frac{x_B \vec{P}_{h\perp}^2}{z_h Q^2} p^+$$

Thus $P_h^\mu = \frac{x_B \vec{P}_{h\perp}^2}{z_h Q^2} p^+ \bar{n}^\mu + \frac{z_h Q^2}{2x_B p^+} n^\mu + P_{hT}^\mu \qquad (P_{hT}^\mu P_{hT\mu} = -\vec{P}_{h\perp}^2)$

$$P_c^\mu = \frac{1}{z} P_h^\mu \qquad \left(P_{c\perp} = \frac{P_{h\perp}}{z}\right)$$

$$= \frac{x_B P_{c\perp}^2}{\hat{z} Q^2} p^+ \bar{n}^\mu + \frac{\hat{z} Q^2}{2x_B p^+} n^\mu + P_{cT}^\mu$$

DIS normalization



from CTEQ handbook

$$E' \frac{d\sigma}{d^3 l'} = \left(\frac{2}{S}\right)\left(\frac{\alpha_{em}}{Q^2}\right)^2 L^{\mu\nu} W_{\mu\nu}$$

where $L^{\mu\nu} = \frac{1}{2} \mathrm{Tr}[\not{l} \gamma^\mu \not{l}' \gamma^\nu]$

$$W_{\mu\nu} = \frac{1}{4\pi} \int d^4 y \; e^{iq\cdot y} \quad \frac{1}{2} \sum_s \langle PS | J^\dagger_\mu(y) \; J_\nu(0) | PS \rangle$$

Note $\frac{d^3 l'}{E'} = \frac{\pi Q^2}{x_B^2 S} \; dx_B \, dQ^2$

$\Downarrow$ define $y \equiv \frac{Q^2}{x_B S}$

$$= \pi S \, y \; dx_B \, dy$$

$$\frac{d\sigma}{dx_B \, dy} = \frac{2\pi \alpha_{em}^2 \, y}{(Q^2)^2} \; L^{\mu\nu} W_{\mu\nu}$$

$\Downarrow$ take $\frac{1}{4\pi}$ out from $W_{\mu\nu}$

$$= \frac{\alpha_{em}^2 \, y}{2(Q^2)^2} \; L^{\mu\nu} W_{\mu\nu}$$

In a so-called hadron frame, one could write

$$\frac{2}{Q^2} L^{\mu\nu} = (1+\cosh^2\psi)(X^\mu X^\nu + Y^\mu Y^\nu) + 2\sinh^2\psi \; T^\mu T^\nu$$

$$\cosh\psi = \frac{2}{y} - 1$$

$$X^\mu X^\nu + Y^\mu Y^\nu = -g^{\mu\nu} + T^\mu T^\nu - Z^\mu Z^\nu$$

$$\Big\Downarrow \qquad T^\mu = \frac{1}{Q}(q^\mu + 2x_B p^\mu)$$

$$Z^\mu = -\frac{q^\mu}{Q}$$

drop all q^μ, q^ν since $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$

$$= -g^{\mu\nu} + \frac{4x_B^2}{Q^2} p^\mu p^\nu$$

Thus

$$\frac{2}{Q^2} L^{\mu\nu} \Rightarrow \frac{2}{y^2}\Big[\underbrace{\Big(-g^{\mu\nu} + \frac{4x_B^2}{Q^2}p^\mu p^\nu\Big)}_{} (1+(1-y)^2) + 2 * \frac{4x_B^2}{Q^2}p^\mu p^\nu (2(1-y))\Big]$$

$\Downarrow$ Called transverse projection — longitudinal projection

$$= \frac{2}{y^2}\Big[(-g^{\mu\nu})(1+(1-y)^2) + \Big(\frac{4x_B^2}{Q^2}p^\mu p^\nu\Big)(1+4(1-y)+(1-y)^2)\Big]$$

refer to "Metric" contribution — longitudinal contribution

See, eg. hep-ph/0411212

If we've only interested in Metric contribution, then well have

$$\frac{2}{Q^2} L^{\mu\nu} \rightarrow \frac{2}{y^2}[1+(1-y)^2](-g^{\mu\nu})$$

Then

$$\frac{d\sigma}{dx_B\,dy} = \frac{\alpha_{em}^2\, y}{2(Q^2)^2}\,\frac{Q^2}{2}\,\frac{2}{y^2}[1+(1-y)^2](-g^{\mu\nu})W_{\mu\nu}$$

$$= \frac{\alpha_{em}^2}{Q^2}\,\frac{1+(1-y)^2}{2y}\,(-g^{\mu\nu})W_{\mu\nu}$$

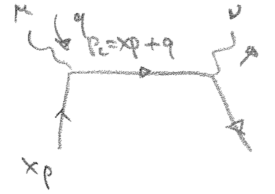
At the partonic level

$$\frac{d\sigma}{dx_B\,dy} = \frac{\alpha_{em}^2}{Q^2}\;\frac{1+(1-y)^2}{2y}\int\frac{dx}{x}\,dz\; f_{q/p}(x)\; D_{q\to h}(z)\left[-g^{\mu\nu}H_{\mu\nu}\right] dps^{(n)}$$

---

for example, at leading order

$$-g^{\mu\nu}H_{\mu\nu} =$$



$$= \frac{1}{2}\mathrm{Tr}\left[(x\not{p})\,\gamma^\nu\,(x\not{p}+\not{q})\,\gamma^\mu\right](-g_{\mu\nu})$$

$$= 4(1-\epsilon)\; xp\cdot q$$

$$= (1-\epsilon)\; 2\,\frac{x}{x_B}\,Q^2$$

$$dps^{(1)} = \frac{d^{n-1}p_c}{(2\pi)^{n-1}2E_c}\;(2\pi)^n\,\delta^n(xp+q-p_c)$$

$$\Big\Downarrow\quad p_c = \frac{1}{z}P_h$$

$$= \frac{1}{z^{n-2}}\;\frac{d^{n-1}P_h}{(2\pi)^{n-1}2E_h}\;(2\pi)^n\,\delta^n(xp+q-p_c)$$

$$= \frac{1}{z^{n-2}}\;\frac{d^nP_h}{\cancel{(2\pi)^n}}\;2\pi\,\delta(P_h^2)\;\cancel{(2\pi)^n}\;\delta^n(xp+q-p_c)$$

$$= \frac{1}{z^{n-2}}\;dP_h^+\,dP_h^-\,d^{n-2}P_{hT}\;\;2\pi\,\underbrace{\delta\left(2P_h^+P_h^- - \vec{P}_{hT}^{\,2}\right)}_{\frac{1}{2P_h^-}\delta\left(P_h^+ - \frac{\vec{P}_{h\perp}^{\,2}}{2P_h^-}\right)}\;\;\delta(xp^+ + q^+)\;\delta(q^- - p_c^-)\;\delta^{n-2}(p_{c\perp})$$

$$\Big\Downarrow\quad \frac{dP_h^-}{P_h^-} = \frac{dz_h}{z_h} \qquad\qquad \delta^{n-2}(p_{cT}) = \delta^{n-2}(P_{hT}/z) = z^{n-2}\,\delta^{n-2}(P_{hT})$$

$$= \frac{1}{z^{n-2}}\;\frac{dz_h}{2z_h}\;d^{n-2}P_{hT}\;\;z^{n-2}\,\delta^{n-2}(P_{hT})\;\frac{1}{p^+}\,\delta(x-x_B)\;\frac{1}{q^-}\,\delta(1-\hat{z})\;*\,2\pi$$

$$dPS^{(1)} = \frac{dz_h}{2z_h} \quad \frac{1}{p^+q^-} \quad \frac{1}{x} \, \delta(1-\hat{x}) \, \delta(1-\hat{z}) \quad * 2\pi$$

$$\Downarrow \qquad 2p^+q^- = 2p\cdot q = \frac{Q^2}{x_B}$$

$$= \frac{dz_h}{z_h} \quad \frac{x_B}{x} \frac{1}{Q^2} \quad \delta(1-\hat{x}) \, \delta(1-\hat{z}) \quad * 2\pi \quad = \quad dz_h \; * \; \frac{x_B}{z \, x \, Q^2} \quad \delta(1-\hat{x}) \, \delta(1-\hat{z}) \; * 2\pi$$

$$\Downarrow \quad z_h = z$$

$$(-g^{\mu\nu}) H_{\mu\nu} \, dPS^{(1)} = 2(1-\epsilon) \, \cancel{\frac{x}{x_B} Q^2} \quad * \quad \frac{dz_h}{z} \quad \cancel{\frac{x_B}{x} \frac{1}{Q^2}} \quad \delta(1-\hat{x}) \, \delta(1-\hat{z}) \quad * 2\pi$$

Thus

$$\frac{d\sigma}{dx_B \, dy \, dz_h} = \frac{2\pi \alpha_{em}^2}{Q^2} \quad \frac{1+(1-y)^2}{2y} \quad * \; 2(1-\epsilon) \int \frac{dx}{x} \frac{dz}{z} \; f_{q/p}(x) \; D_{q\to h}(z)$$

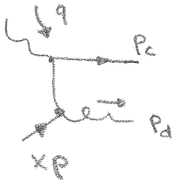
$$* \; \delta(1-\hat{x}) \, \delta(1-\hat{z})$$

define $\sigma_0 = \frac{2\pi \alpha_{em}^2}{Q^2} \; \frac{1+(1-y)^2}{y} \; (1-\epsilon)$, then

$$\frac{d\sigma}{dx_B \, dy \, dz_h} = \sigma_0 \int \frac{dx}{x} \; \frac{dz}{z} \quad f_{q/p}(x) \; D_{q\to h}(z) \quad * \quad \delta(1-\hat{x}) \, \delta(1-\hat{z})$$

Study higher order

Now for real diagram, we have $dps^{(2)}$



$$dps^{(2)} = \frac{d^{n-1}P_c}{(2\pi)^{n-1}2E_c}\frac{d^{n-1}P_d}{(2\pi)^{n-1}2E_d}(2\pi)^n\delta^n(xp+q-P_c-P_d)$$

$$= \frac{d^{n-1}P_h}{(2\pi)^{n-1}2E_h}\frac{1}{z^{n-2}}\frac{d^nP_d}{\cancel{(2\pi)^d}}2\pi\delta(P_d^2) * \cancel{(2\pi)^n}\delta^n(xp+q-P_c-P_d)$$

$$= \frac{d^nP_h}{(2\pi)^n}2\pi\delta(P_h^2)\frac{1}{z^{n-2}}2\pi\delta(P_d^2)$$

$$= dP_h^+dP_h^-d^{n-2}P_{h\perp}\underbrace{\delta(2P_h^+P_h^- - \vec{P}_{h\perp}^2)}_{\frac{1}{2P_h^-}\delta\left(P_h^+ - \frac{\vec{P}_{h\perp}^2}{2P_h^-}\right)}\frac{1}{z^{n-2}(2\pi)^{n-2}}\delta\left[(xp+q-P_c)^2\right]$$

$$\Downarrow\quad \frac{dP_h^-}{P_h^-} = \frac{dz_h}{z_h}$$

$$= \frac{dz_h}{2z_h}d^{n-2}P_{h\perp}\frac{1}{(2\pi z)^{n-2}}\delta\left[(xp+q-P_c)^2\right]$$

$$(xp+q-P_c)^2 = (xp+q)^2 - 2P_c\cdot(xp+q)$$

$$= -Q^2 + x2P\cdot q - x2P_c\cdot P - 2P_c\cdot q$$

define $\hat{S} = (xp+q)^2 = -Q^2 + x2P\cdot q = -Q^2 + x\frac{Q^2}{x_B} \overset{\hat{x}=\frac{x_B}{x}}{=} \frac{Q^2(1-\hat{x})}{\hat{x}}$

$$\hat{t} = (P_c-q)^2 = -Q^2 - 2P_c\cdot q = -Q^2 - [2P_c^+q^- + 2P_c^-q^+]$$

$$= -Q^2 - \left[2\frac{x_B P_{c\perp}^2}{\hat{z}Q^2}p^+\frac{Q^2}{2x_Bp^+} + 2\frac{\hat{z}Q^2}{2x_Bp^+}(-x_Bp^+)\right]$$

$$= -Q^2 - \left[\frac{P_{c\perp}^2}{\hat{z}} - \hat{z}Q^2\right]$$

$$= -\left[(1-\hat{z})Q^2 + \frac{P_{c\perp}^2}{\hat{z}}\right]$$

$$\hat u = (xp - p_c)^2 = x\,(-2p\cdot p_c) = x\,(-2)\,p^+ \,\frac{\hat z\, Q^2}{2x_B p^+} = -\frac{\hat z}{\hat x}\, Q^2$$

Thus from $\quad 0 = (xp+q-p_c)^2$

$$\Rightarrow \quad \delta\left[(xp+q-p_c)^2\right] = \delta\left[\hat s+\hat t+\hat u+Q^2\right]$$

$$= \delta\left[\frac{Q^2(1-\hat x)}{\hat x} - (1-\hat z)\,Q^2 - \frac{p_{c\perp}^2}{\hat z} - \frac{\hat z}{\hat x}\,Q^2 + Q^2\right]$$

$$= \delta\left[\frac{p_{c\perp}^2}{\hat z} - \frac{Q^2}{\hat x}(1-\hat x)(1-\hat z)\right]$$

$$= \hat z\;\delta\left[p_{c\perp}^2 - Q^2\,\frac{\hat z(1-\hat z)(1-\hat x)}{\hat x}\right]$$

Thus $\quad p_{c\perp}^2 = \dfrac{Q^2\,\hat z(1-\hat z)(1-\hat x)}{\hat x}$

$\Rightarrow$ Thus $\quad \hat t = -\left[(1-\hat z)\,Q^2 + \dfrac{\vec p_{c\perp}^{\,2}}{\hat z}\right] = -\left[(1-\hat z)\,Q^2 + Q^2(1-\hat z)\,\dfrac{(1-\hat x)}{\hat x}\right]$

$$= -\left[Q^2(1-\hat z)\,\frac{1}{\hat x}\right]$$

$$\boxed{\begin{aligned} \hat s &= \frac{1-\hat x}{\hat x}\,Q^2 \\ \hat t &= -\frac{1-\hat z}{\hat x}\,Q^2 \\ \hat u &= -\frac{\hat z}{\hat x}\,Q^2 \end{aligned}}$$

$$\frac{\hat t\,\hat u\,\hat s}{(\hat s+Q^2)^2} = \frac{\frac{1-\hat z}{\hat x}\,Q^2 * \frac{\hat z}{\hat x}\,Q^2 * \frac{1-\hat x}{\hat x}\,Q^2}{\left(\frac{Q^2}{\hat x}\right)^2}$$

$$= \frac{\hat z(1-\hat z)(1-\hat x)}{\hat x}\,Q^2 = p_{c\perp}^2$$

$$\Rightarrow \quad \boxed{p_{c\perp}^2 = \frac{\hat t\,\hat u\,\hat s}{(\hat s+Q^2)^2}}$$

$$\boxed{p_{h\perp}^2 = z^2\,p_{c\perp}^2 = z^2\,\frac{\hat t\,\hat u\,\hat s}{(\hat s+Q^2)^2}}$$

$$\delta[(xp+q-p_c)^2] = \hat{z}\,\delta\left[\vec{p}_{c\perp}^{\,2} - \frac{Q^2\hat{z}(1-\hat{z})(1-\hat{x})}{\hat{x}}\right]$$

$$\Downarrow \quad p_{c\perp}^2 = \frac{p_{h\perp}^2}{z^2}$$

$$= \hat{z}\, z^2\, \delta\left[p_{h\perp}^2 - \frac{z^2 Q^2 \hat{z}(1-\hat{z})(1-\hat{x})}{\hat{x}}\right]$$

Thus

$$dps^{(2)} = \frac{dz_h}{2z_h}\, d^{n-2}p_{h\perp}\, \frac{1}{(2\pi z)^{n-2}}\quad \hat{z}\, z^2\, \delta\left[p_{h\perp}^2 - \frac{z^2Q^2\hat{z}(1-\hat{z})(1-\hat{x})}{\hat{x}}\right]$$

Note $\int d^d p_{h\perp} = \int p_{h\perp}^{d-1}\, dp_{h\perp} * \Omega_d$

$$= \frac{1}{2}\,(p_{h\perp}^2)^{\frac{d-2}{2}}\, dp_{h\perp}^2 * \frac{2\pi^{d/2}}{\Gamma(d/2)}$$

$$= \frac{\pi^{d/2}}{\Gamma(d/2)}\,(p_{h\perp}^2)^{\frac{d-2}{2}}\, dp_{h\perp}^2$$

$$\Downarrow \quad d = n-2 = 2-2\epsilon$$

$$= \frac{\pi^{1-\epsilon}}{\Gamma(1-\epsilon)}\,(p_{h\perp}^2)^{-\epsilon}\, dp_{h\perp}^2$$

$$dps^{(2)} = \frac{dz_h}{2z_h} * \frac{\pi^{1-\epsilon}}{\Gamma(1-\epsilon)}\,(p_{h\perp}^2)^{-\epsilon}\, dp_{h\perp}^2\, \frac{1}{(2\pi z)^{2-2\epsilon}} * \hat{z}\, z^2$$
$$* \,\delta\left[p_{h\perp}^2 - \frac{z^2Q^2\hat{z}(1-\hat{z})(1-\hat{x})}{\hat{x}}\right]$$

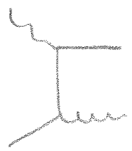
$$= \left(dz_h\, \frac{1}{z}\right) * \frac{1}{8\pi}\left(\frac{4\pi}{Q^2}\right)^{\epsilon}\frac{1}{\Gamma(1-\epsilon)}\left[\hat{z}(1-\hat{z})\right]^{-\epsilon}\left[(1-\hat{x})^{-\epsilon}\,\hat{x}^{\epsilon}\right]$$

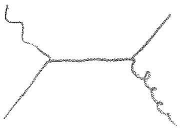
Thus for spin-averaged one

$$\frac{d\sigma}{dx_B\, dy\, dz_h} = \frac{\alpha_{em}^2}{Q^2} \frac{1+(1-y)^2}{2y} \int \frac{dx}{x} \frac{dz}{z} f_{q/p}(x)\, D_{q\to h}(z) \left[-g^{\mu\nu} H_{\mu\nu}\right]$$

$$* \frac{1}{8\pi} \left(\frac{4\pi}{Q^2}\right)^{\epsilon} \frac{1}{\Gamma(1-\epsilon)} \hat{z}^{-\epsilon} (1-\hat{z})^{-\epsilon} \quad \hat{x}^{\epsilon} (1-\hat{x})^{-\epsilon}$$

Let's study unpolarized cross-section first





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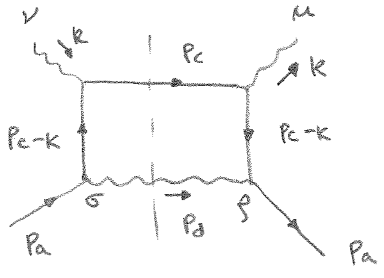
①



$P_d = k + P_a - P_c$

$k^2 = -Q^2$

define $\hat{s} = (P_a + k)^2 = -Q^2 + 2P_a \cdot k$

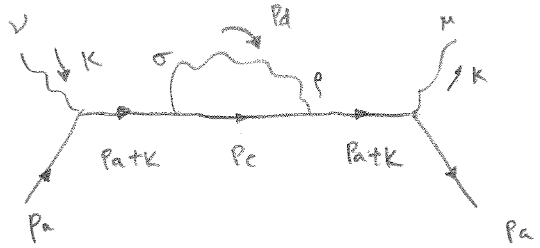
$\hat{t} = (P_c - k)^2 = -Q^2 - 2P_c \cdot k$

$\hat{u} = (P_a - P_c)^2 = -2P_a \cdot P_c$

$$\text{Fig1} = \frac{1}{2}\,\mathrm{Tr}\left[\not{P}_a\, \gamma^\rho (\not{P}_c - \not{k})\, \gamma^\mu \not{P}_c\, \gamma^\nu (\not{P}_c - \not{k})\, \gamma^\sigma\right] (-g_{\mu\nu})\, d_{\rho\sigma}(P_d)$$

$$* \left[\frac{1}{(P_c - k)^2}\right]^2 * g_s^2$$

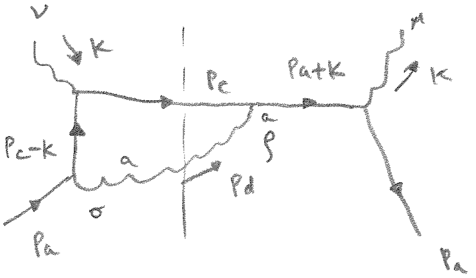
②



$$\text{Fig2} = \frac{1}{2}\,\mathrm{Tr}\left[\not{P}_a\, \gamma^\mu (\not{P}_a + \not{k})\, \gamma^\rho \not{P}_c\, \gamma^\sigma (\not{P}_a + \not{k})\, \gamma^\nu\right] (-g_{\mu\nu})\, d_{\rho\sigma}(P_d)$$

$$* \left[\frac{1}{(P_a + k)^2}\right]^2 * g_s^2$$

③

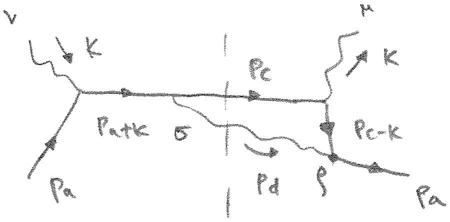


color = $\frac{1}{N}\,\mathrm{Tr}[T^a T^a] = C_F$

$$\text{Fig3} = \frac{1}{2}\,\mathrm{Tr}[\not{P}_a \gamma^\mu (\not{P}_a+\not{k}) \gamma^\rho \not{P}_c \gamma^\nu (\not{P}_c-\not{k}) \gamma^\sigma]\,(-g_{\mu\nu})\; d_{\rho\sigma}(P_d)$$

$$* \frac{1}{(P_c-k)^2}\;\frac{1}{(P_a+k)^2}$$

④



$$\text{Fig4} = \frac{1}{2}\,\mathrm{Tr}[\not{P}_a \gamma^\rho (\not{P}_c-\not{k}) \gamma^\mu \not{P}_c \gamma^\sigma (\not{P}_a+\not{k}) \gamma^\nu]\,(-g_{\mu\nu})\; d_{\rho\sigma}(P_d)$$

$$* \frac{1}{(P_c-k)^2}\;\frac{1}{(P_a+k)^2}$$

$$\text{Fig}1+2+3+4 = 4(1-\epsilon)\,\frac{1}{\hat{s}\hat{t}}\left[-(1-\epsilon)(\hat{s}^2+\hat{t}^2) + 2\epsilon\hat{s}\hat{t} - 2Q^2(\underbrace{Q^2+\hat{s}+\hat{t}}_{-\hat{u}})\right]$$

$$= 4(1-\epsilon)\left[(1-\epsilon)\left(\frac{\hat{s}}{-\hat{t}} + \frac{-\hat{t}}{\hat{s}}\right) + \frac{2Q^2\hat{u}}{\hat{s}\hat{t}} + 2\epsilon\right]$$

Eventually we have

$$\frac{d\sigma}{dx_B\,dy\,dz_h} = \frac{\alpha_{em}^2}{Q^2}\,\frac{1+(1-y)^2}{2y}\int\frac{dx}{x}\,\frac{dz}{z}\,f_{q/p}(x)\,D_{q\to h}(z)$$

$$* \,(g_s\mu^\epsilon)^2 * 4(1-\epsilon)\left[(1-\epsilon)\left(-\frac{\hat{s}}{\hat{t}} - \frac{\hat{t}}{\hat{s}}\right) + \frac{2Q^2\hat{u}}{\hat{s}\hat{t}} + 2\epsilon\right]$$

$$* \,\frac{1}{8\pi}\left(\frac{4\pi}{Q^2}\right)^\epsilon\frac{1}{\Gamma(1-\epsilon)}\;\hat{z}^{-\epsilon}(1-\hat{z})^{-\epsilon}\;\hat{x}^{\epsilon}(1-\hat{x})^{-\epsilon}$$

$$= \frac{2\pi\alpha_{em}^2}{Q^2}\,\frac{1+(1-y)^2}{y} * \frac{\alpha_s}{2\pi}\int\frac{dx}{x}\,\frac{dz}{z}\,f_{q/p}(x)\,D_{q\to h}(z)$$

$$* \left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon\frac{1}{\Gamma(1-\epsilon)}\;\hat{z}^{-\epsilon}(1-\hat{z})^{-\epsilon}\;\hat{x}^{\epsilon}(1-\hat{x})^{-\epsilon}$$

$$* \,(1-\epsilon)\left[(1-\epsilon)\left(-\frac{\hat{s}}{\hat{t}} - \frac{\hat{t}}{\hat{s}}\right) + \frac{2Q^2\hat{u}}{\hat{s}\hat{t}} + 2\epsilon\right]$$

define (like before) $\sigma_0 = \frac{2\pi\alpha_{em}^2}{Q^2}\,\frac{1+(1-y)^2}{y}\,(1-\epsilon)$

$\frac{dx}{x} = \frac{d\hat{x}}{\hat{x}}$ $\quad\frac{dz}{z} = \frac{d\hat{z}}{\hat{z}}$ $\quad\boxed{\text{Color} = C_F}$

$$\frac{d\sigma}{dx_B\,dy\,dz_h} = \sigma_0\,\frac{\alpha_s}{2\pi}\int\frac{dx}{x}\,\frac{dz}{z}\,f_{q/p}(x)\,D_{q\to h}(z)$$

$$* \left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon\frac{1}{\Gamma(1-\epsilon)}\;\hat{z}^{-\epsilon}(1-\hat{z})^{-\epsilon}\;\hat{x}^{\epsilon}(1-\hat{x})^{-\epsilon}$$

$$* \left[(1-\epsilon)\left(-\frac{\hat{s}}{\hat{t}} - \frac{\hat{t}}{\hat{s}}\right) + \frac{2Q^2\hat{u}}{\hat{s}\hat{t}} + 2\epsilon\right]$$

$\hat{s} = \frac{1-\hat{x}}{\hat{x}} Q^2 \qquad \hat{t} = -\frac{1-\hat{z}}{\hat{x}} Q^2 \qquad \hat{u} = -\frac{\hat{z}}{\hat{x}} Q^2$

$$[\cdots] = \left\{ (1-\epsilon)\left[\frac{1-\hat{x}}{1-\hat{z}} + \frac{1-\hat{z}}{1-\hat{x}} \right] + \frac{2\hat{x}}{1-\hat{x}} \frac{\hat{z}}{1-\hat{z}} + 2\epsilon \right\}$$

$$\frac{d\sigma}{dx_B\, dy\, dz_h} = \sigma_0 \frac{\alpha_s}{2\pi} \int \frac{dx}{x} \frac{dz}{z} f_{q/p}(x)\, D_{q\to h}(z)$$

$$* \left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \hat{z}^{-\epsilon}(1-\hat{z})^{-\epsilon} \hat{x}^{\epsilon} (1-\hat{x})^{-\epsilon}$$

$$* \left[(1-\epsilon)\left(\frac{1-\hat{x}}{1-\hat{z}} + \frac{1-\hat{z}}{1-\hat{x}} \right) + \frac{2\hat{x}}{1-\hat{x}} \frac{\hat{z}}{1-\hat{z}} + 2\epsilon \right]$$

$$\hat{z}^{-\epsilon}(1-\hat{z})^{-\epsilon-1} = -\frac{1}{\epsilon}\delta(1-\hat{z}) + \frac{1}{(1-\hat{z})_+} - \epsilon\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ - \epsilon \frac{\ln\hat{z}}{1-\hat{z}} + O(\epsilon^2)$$

$$\hat{x}^{\epsilon}(1-\hat{x})^{1-\epsilon} = (1-\hat{x})\left[1 + \epsilon \ln\frac{\hat{x}}{1-\hat{x}}\right]$$

$$\hat{z}^{-\epsilon}(1-\hat{z})^{1-\epsilon} = (1-\hat{z})\left[1 - \epsilon(\ln\hat{z} + \ln(1-\hat{z}))\right]$$

$$\hat{x}^{\epsilon}(1-\hat{x})^{-\epsilon-1} = -\frac{1}{\epsilon}\delta(1-\hat{x}) + \frac{1}{(1-\hat{x})_+} - \epsilon\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ - \epsilon\frac{\ln\hat{x}}{1-\hat{x}} + O(\epsilon^2)$$

$$\hat{z}^{1-\epsilon}(1-\hat{z})^{-\epsilon-1} = -\frac{1}{\epsilon}\delta(1-\hat{z}) + \frac{\hat{z}}{(1-\hat{z})_+} - \epsilon\, \hat{z}\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ - \epsilon\frac{\hat{z}}{1-\hat{z}}\ln\hat{z}$$

$$\hat{x}^{1+\epsilon}(1-\hat{x})^{-\epsilon-1} = -\frac{1}{\epsilon}\delta(1-\hat{x}) + \frac{\hat{x}}{(1-\hat{x})_+} - \epsilon\, \hat{x}\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ + \epsilon\frac{\hat{x}}{1-\hat{x}}\ln\hat{x}$$

$$\hat{z}^{-\epsilon}(1-\hat{z})^{-\epsilon} = 1 - \epsilon(\ln\hat{z} + \ln(1-\hat{z}))$$

$$\hat{x}^{\epsilon}(1-\hat{x})^{-\epsilon} = 1 + \epsilon\ln\frac{\hat{x}}{1-\hat{x}}$$

$$\frac{d\sigma}{dx_B\,dy\,dz_h} = \sigma_0 \frac{\alpha_s}{2\pi} \int \frac{dx}{x}\frac{dz}{z}\; f_{q/p}(x)\; D_{q\to h}(z) \left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon} \frac{1}{\Gamma(1-\epsilon)}$$

$$*\Big\{(1-\epsilon)\left[-\frac{1}{\epsilon}\delta(1-\hat{z}) + \frac{1}{(1-\hat{z})_+}\right]\left[1+\epsilon\ln\frac{\hat{x}}{1-\hat{x}}\right](1-\hat{x})$$

$$+(1-\epsilon)(1-\hat{z})\left[1-\epsilon\ln\hat{z}(1-\hat{z})\right]\left[-\frac{1}{\epsilon}\delta(1-\hat{x}) + \frac{1}{(1-\hat{x})_+}\right]$$

$$+2\left[-\frac{1}{\epsilon}\delta(1-\hat{x}) + \frac{\hat{\lambda}}{(1-\hat{x})_+} - \epsilon\hat{x}\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ + \epsilon\hat{x}\frac{\ln\hat{x}}{1-\hat{x}}\right]$$

$$*\left[-\frac{1}{\epsilon}\delta(1-\hat{z}) + \frac{\hat{z}}{(1-\hat{z})_+} - \epsilon\hat{z}\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ - \epsilon\hat{z}\frac{\ln\hat{z}}{1-\hat{z}}\right]$$

$$+2\epsilon\Big\}$$

$$\{\ldots\} = (1-\hat{x})\left[-\frac{1}{\epsilon}\underset{\blacksquare}{\delta(1-\hat{z})} + \underset{\sim\sim}{\frac{1}{(1-\hat{z})_+}} + \left(1-\ln\frac{\hat{x}}{1-\hat{x}}\right)\underset{=}{\delta(1-\hat{z})}\right]$$

$$+(1-\hat{z})\left[-\frac{1}{\epsilon}\underset{\blacktriangle}{\delta(1-\hat{x})} + \underset{\sim\sim}{\frac{1}{(1-\hat{x})_+}} + \left(1+\ln\hat{z}(1-\hat{z})\right)\underset{\sim}{\delta(1-\hat{x})}\right]$$

$$+2\left[\frac{1}{\epsilon^2}\delta(1-\hat{x})\delta(1-\hat{z}) - \frac{1}{\epsilon}\underset{\blacktriangle}{\delta(1-\hat{x})\frac{\hat{z}}{(1-\hat{z})_+}} - \frac{1}{\epsilon}\underset{\blacksquare}{\delta(1-\hat{z})\frac{\hat{x}}{(1-\hat{x})_+}}\right.$$

$$+\underset{\sim\sim}{\frac{\hat{x}\hat{z}}{(1-\hat{x})_+(1-\hat{z})_+}} + \delta(1-\hat{z})\left(\hat{x}\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ \underset{=}{-}\hat{x}\frac{\ln\hat{x}}{1-\hat{x}}\right)$$

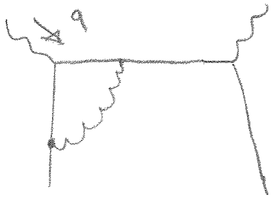
$$\left. + \delta(1-\hat{x})\left(\hat{z}\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ \underset{\sim}{+}\hat{z}\frac{\ln\hat{z}}{1-\hat{z}}\right)\right]$$

$$= \boxed{\frac{2}{\epsilon^2}\delta(1-\hat{x})\delta(1-\hat{z})} - \frac{1}{\epsilon}\delta(1-\hat{x})\frac{1+\hat{z}^2}{(1-\hat{z})_+} - \frac{1}{\epsilon}\delta(1-\hat{z})\frac{1+\hat{x}^2}{(1-\hat{x})_+}$$

$$+\frac{1+(1-\hat{x}-\hat{z})^2}{(1-\hat{x})_+(1-\hat{z})_+} + \delta(1-\hat{z})\left[(1-\hat{x})\left(1-\ln\frac{\hat{x}}{1-\hat{x}}\right) + 2\hat{x}\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ - 2\hat{x}\frac{\ln\hat{x}}{1-\hat{x}}\right]$$

$$+\delta(1-\hat{x})\left[(1-\hat{z})\left(1+\ln\hat{z}(1-\hat{z})\right) + 2\hat{z}\left(\frac{\ln 1-\hat{z}}{1-\hat{z}}\right)_+ + 2\hat{z}\frac{\ln\hat{z}}{1-\hat{z}}\right]$$

Now for virtual diagram



$+ \cdots$

$$\Gamma^{\mu}(q) = \gamma^{\mu} \left\{ 1 + \frac{\alpha_s}{4\pi} C_F \left(\frac{4\pi\mu^2}{-q^2} \right)^{\epsilon} \frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 \right) \right\}$$

2*Re(virtual * lowest order)

$$\Rightarrow \quad \frac{\alpha_s}{2\pi} C_F \left(\frac{4\pi\mu^2}{Q^2} \right)^{\epsilon} \frac{1}{\Gamma(1-\epsilon)}$$

$$* \quad \underbrace{\frac{\Gamma(1+\epsilon)\Gamma^3(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 \right)}_{= \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 \right)}$$

$$\Downarrow$$

$$= \frac{\alpha_s}{2\pi} C_F \left(\frac{4\pi\mu^2}{Q^2} \right)^{\epsilon} \frac{1}{\Gamma(1-\epsilon)} \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 \right]$$

Note $$2\hat{x}\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ = \left[1+\hat{x}^2-(1-\hat{x})^2\right]\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+$$

$$= (1+\hat{x}^2)\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ - (1-\hat{x})\ln(1-\hat{x})$$

---

likewise for $\hat{z}$, we thus have (Real + virtual)

$$\frac{d\sigma}{dx_B\,dy\,dz_h} = \sigma_0 \frac{\alpha_s}{2\pi}\int \frac{dx}{x}\frac{dz}{z} f_{q/p}(x)\, D_{q\to h}(z)\left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \frac{1}{\Gamma(1-\epsilon)}$$

$$* \Bigg[\Big\{ -\frac{1}{\epsilon}\delta(1-\hat{x})\, C_F\left[\frac{1+\hat{z}^2}{(1-\hat{z})_+}+\frac{3}{2}\delta(1-\hat{z})\right]$$

$$-\frac{1}{\epsilon}\delta(1-\hat{z})\, C_F\left[\frac{1+\hat{x}^2}{(1-\hat{x})_+}+\frac{3}{2}\delta(1-\hat{x})\right]\Big\}$$

$$+ C_F\Big\{ \frac{1+(1-\hat{x}-\hat{z})^2}{(1-\hat{x})_+(1-\hat{z})_+}$$

$$+\delta(1-\hat{z})\left[(1+\hat{x}^2)\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ - \frac{1+\hat{x}^2}{1-\hat{x}}\ln\hat{x} + (1-\hat{x})\right]$$

$$+\delta(1-\hat{x})\left[(1+\hat{z}^2)\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ + \frac{1+\hat{z}^2}{1-\hat{z}}\ln\hat{z} + (1-\hat{z})\right]$$

$$-8\,\delta(1-\hat{x})\,\delta(1-\hat{z})\Big\}\Bigg]$$

This result is consistent with NPB160 (1979) 301

Altarelli - Ellis - Martinelli - Pi

(after convert DIS scheme to $\overline{\text{MS}}$ scheme)

Expansion

$$\hat{z}^{-\epsilon}(1-\hat{z})^{-\epsilon-1} = -\frac{1}{\epsilon}\delta(1-\hat{z}) + \frac{1}{(1-\hat{z})_+} - \epsilon\left(\frac{\ln(1-\hat{z})}{1-\hat{z}}\right)_+ - \epsilon\frac{\ln\hat{z}}{1-\hat{z}}$$

$$\hat{x}^{\epsilon}(1-\hat{x})^{-\epsilon-1} = -\frac{1}{\epsilon}\delta(1-\hat{x}) + \frac{1}{(1-\hat{x})_+} - \epsilon\left(\frac{\ln(1-\hat{x})}{1-\hat{x}}\right)_+ + \epsilon\frac{\ln\hat{x}}{1-\hat{x}}$$

$$\hat{x}^{\epsilon}(1-\hat{x})^{-\epsilon} = 1 + \epsilon\ln\frac{\hat{x}}{1-\hat{x}}$$

$$\hat{z}^{-\epsilon}(1-\hat{z})^{-\epsilon} = 1 - \epsilon\ln\hat{z} - \epsilon\ln(1-\hat{z})$$