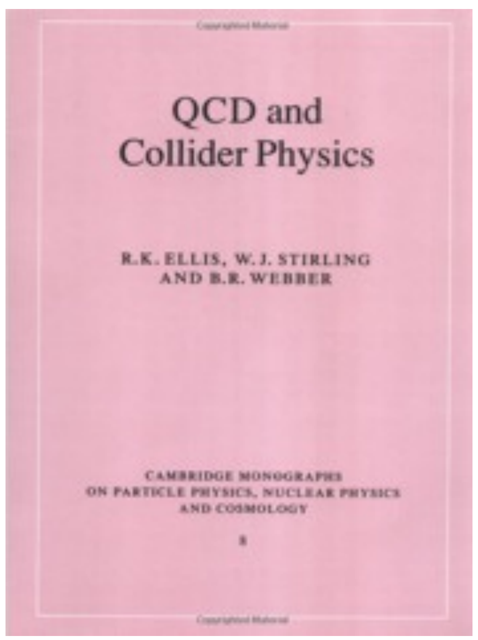
Introduction to pQCD and Jets: lecture 2

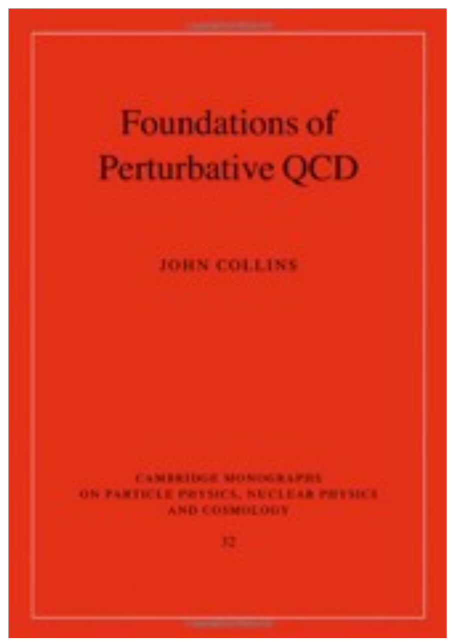
Zhong-Bo Kang
Los Alamos National Laboratory

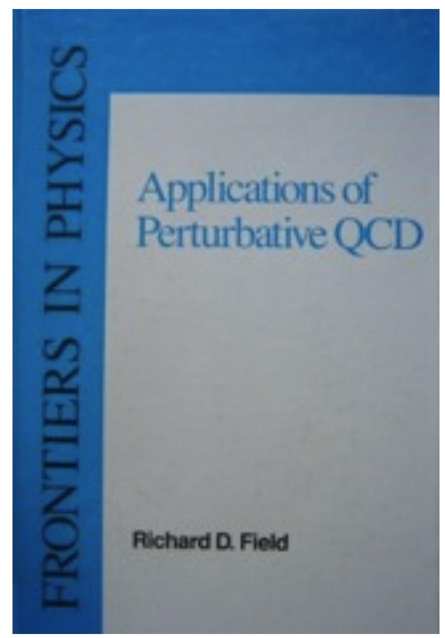
Jet Collaboration Summer School
University of California, Davis
July 19-21, 2014

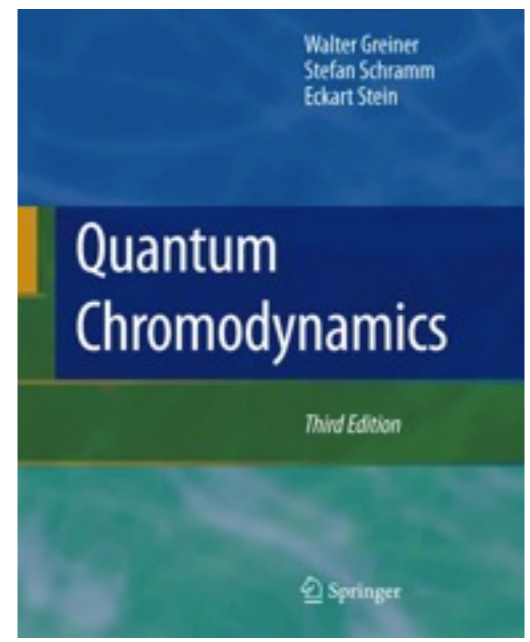
Selected references on QCD

- QCD and Collider Physics: Ellis-Stirling-Webber
- Foundations of Perturbative QCD: J. Collins
- Applications of Perturbative QCD: R. Field
- Quantum Chromodynamics: Greiner-Schramm-Stein









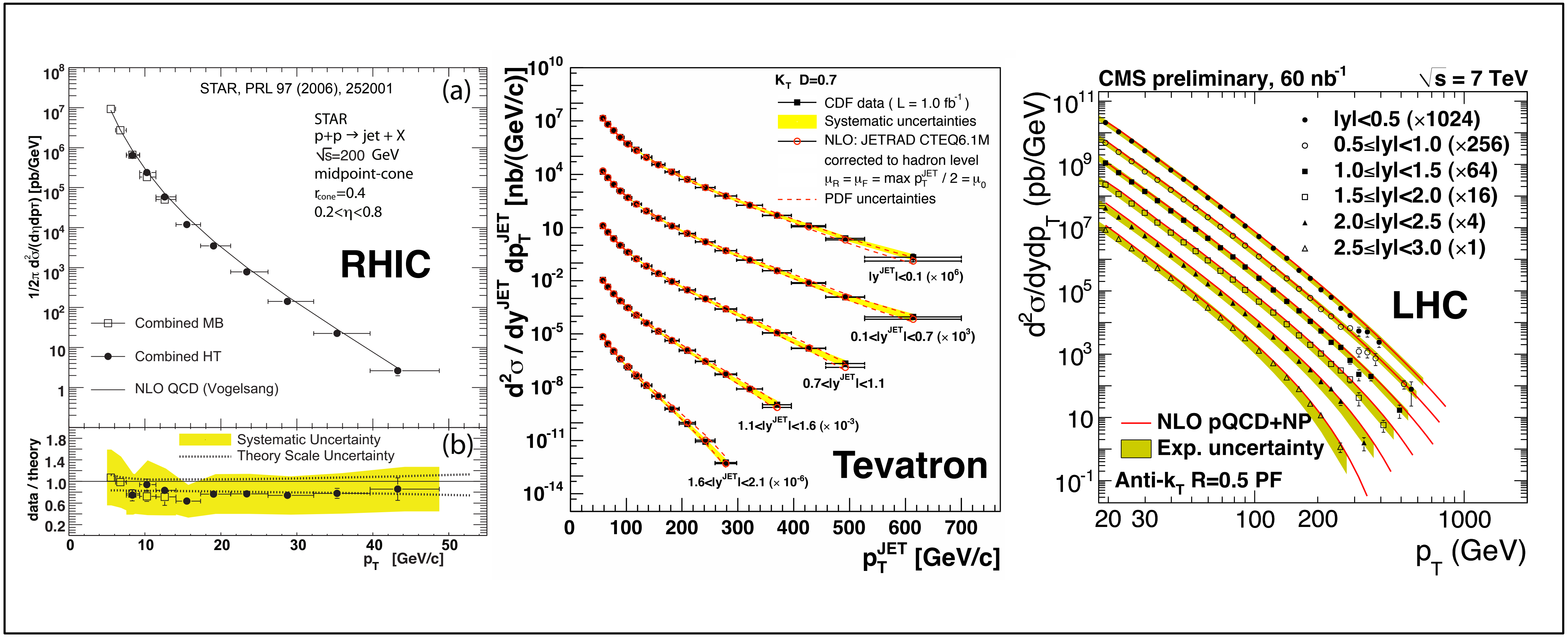
- CTEQ collaboration: http://www.phys.psu.edu/~cteq
- QCD Resource Letter: arXiv:1002.5032 by Kronfeld-Quigg
- Particle Data Group: http://pdg.lbl.gov

Recap - lecture 1: QCD foundation

- pQCD provides a way to extract information on hadron structure and/or probe the properties of QGP
  - Asymptotic freedom: allow one to calculate partonic cross sections
  - Parton distribution functions
  - Renormalization: redefine the coupling constant to absorb the UV divergence, thus the coupling constant now depends on renormalization scale. The scale dependence can be calculated, which gives the renormalization group equation for the coupling constant and exhibits the feature of asymptotic freedom.
  - Factorization: redefine the PDFs to absorb the CO divergence, thus the PDFs now depend on factorization scale. The scale dependence can be calculated, which gives the DGLAP evolution equation for PDFs.

Collinear factorization

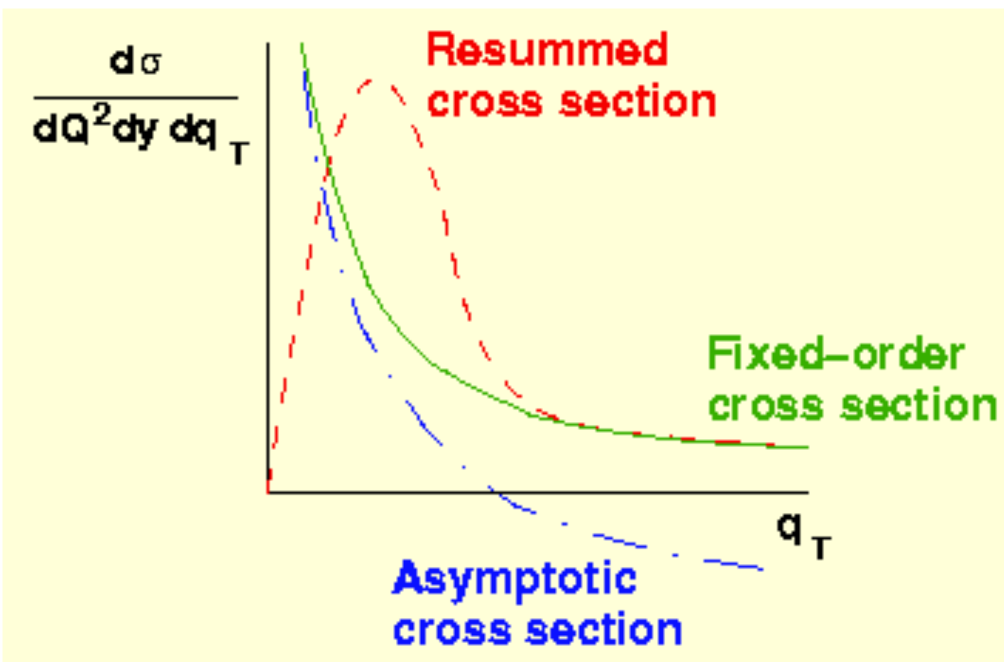
- So far the pQCD factorization we have talked about is the so-called "collinear factorization" formalism, which describes the data very well

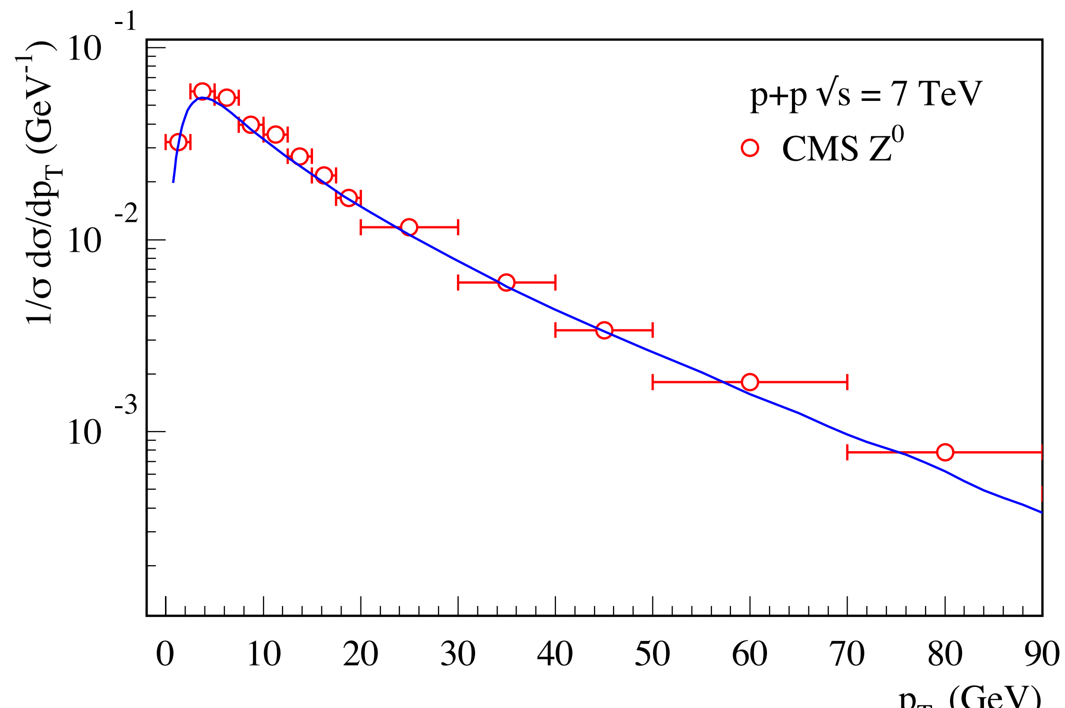


- It is good when there is only "one large momentum" scale in the process, e.g., hadron/jet transverse momentum

TMD factorization

- There are situations where such a formalism breaks down (cannot describe the cross section any more)
  - When there are two/more distinctive momentum scales: e.g., W/Z boson production at low pt (M >> pt) Kang-Qiu, PLB 2013, Echevarria-Idilbi-Kang-Vitev, PRD 2014



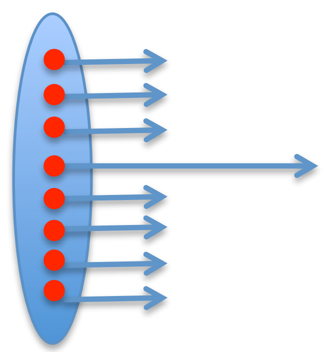


- In the small pt region, the process starts to become sensitive to the parton intrinsic transverse momentum. Another type of factorization will naturally appear - the so-called "TMD factorization" formalism
  - TMD = Transverse Momentum Dependent (distribution and fragmentation function)

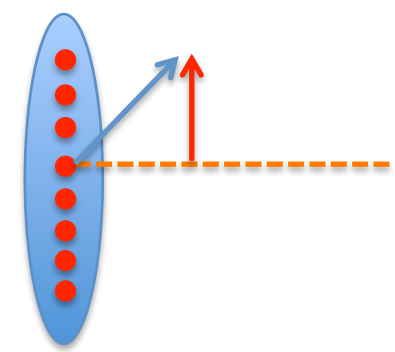
It is time to go beyond collinear picture

- TMD = Parton distribution with a transverse component

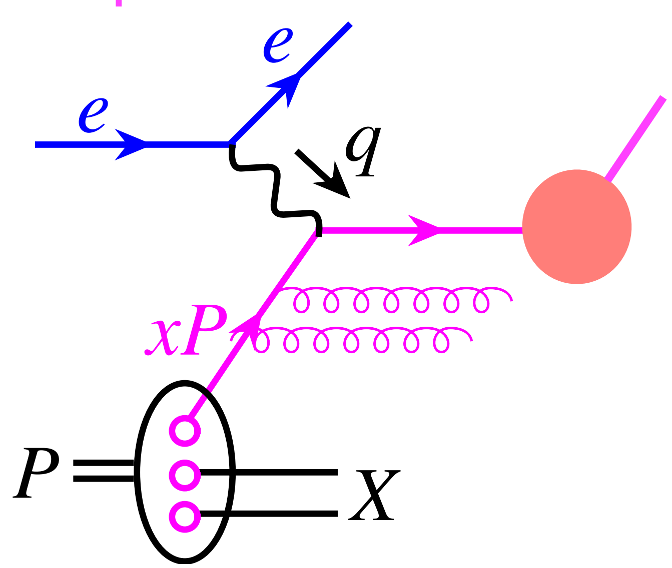
$f(x)$ $\qquad\qquad$ $f(x, k_\perp)$



$p_a = x_a P_A$



- In perturbation theory order by order, you see the appearance of large logarithms
  - One can trace back to understand these large logarithms from the kt-dependent parton distribution function (TMD evolution or resummation)



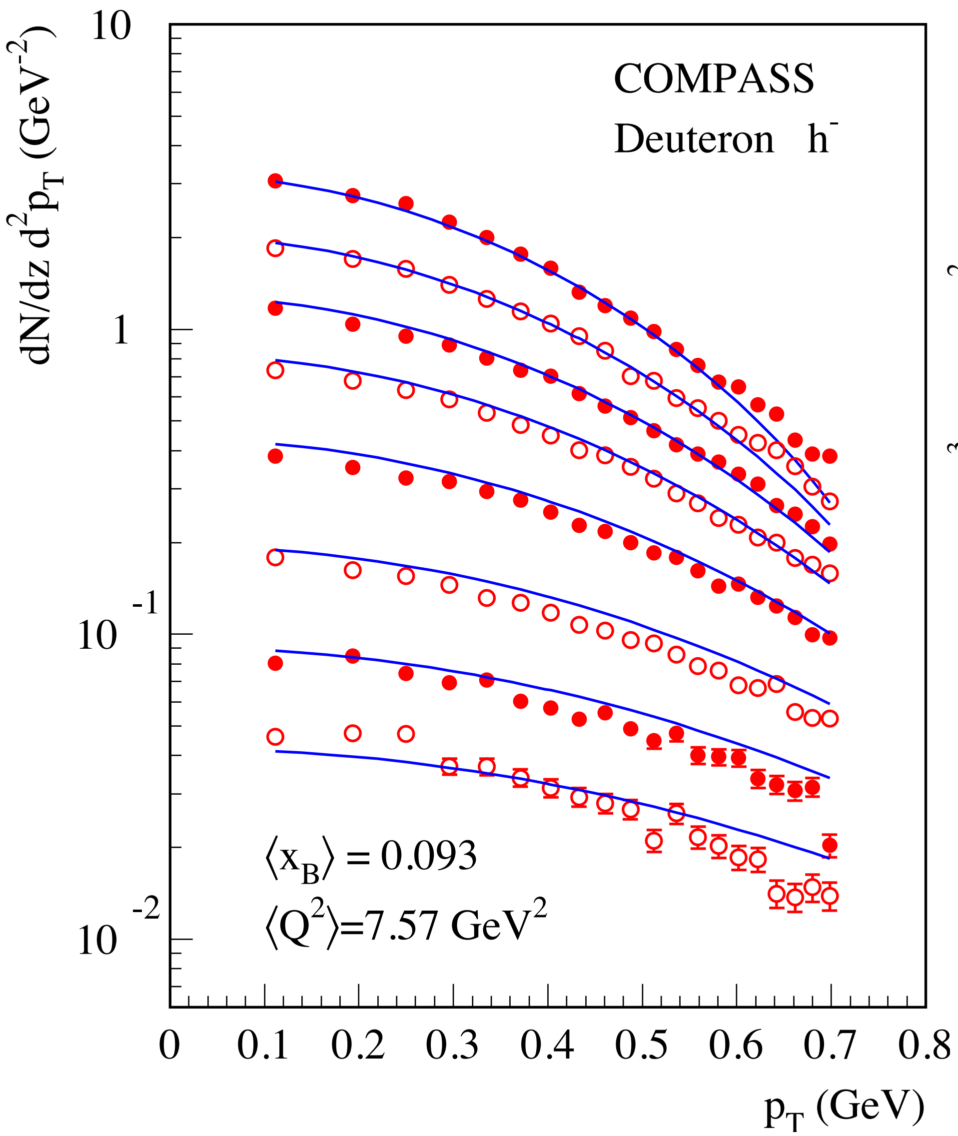
$$\left(\alpha_s \ln^2 \frac{Q^2}{q_T^2}\right)^n$$

TMD evolution and resummation

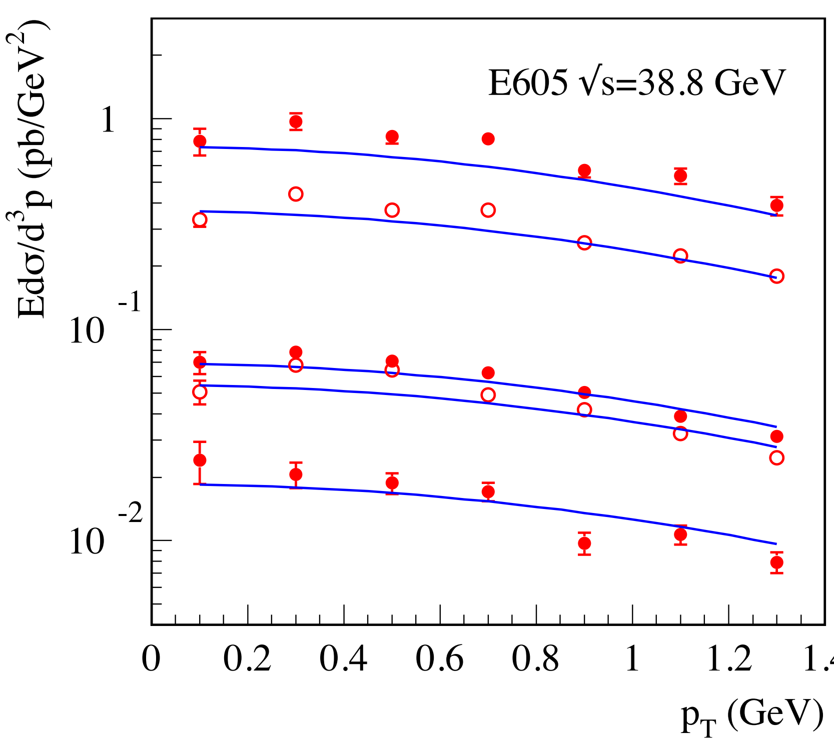
- This is the most tough business in the current QCD development, there is a workshop dedicated to this particular field (Annual QCD evolution workshop)

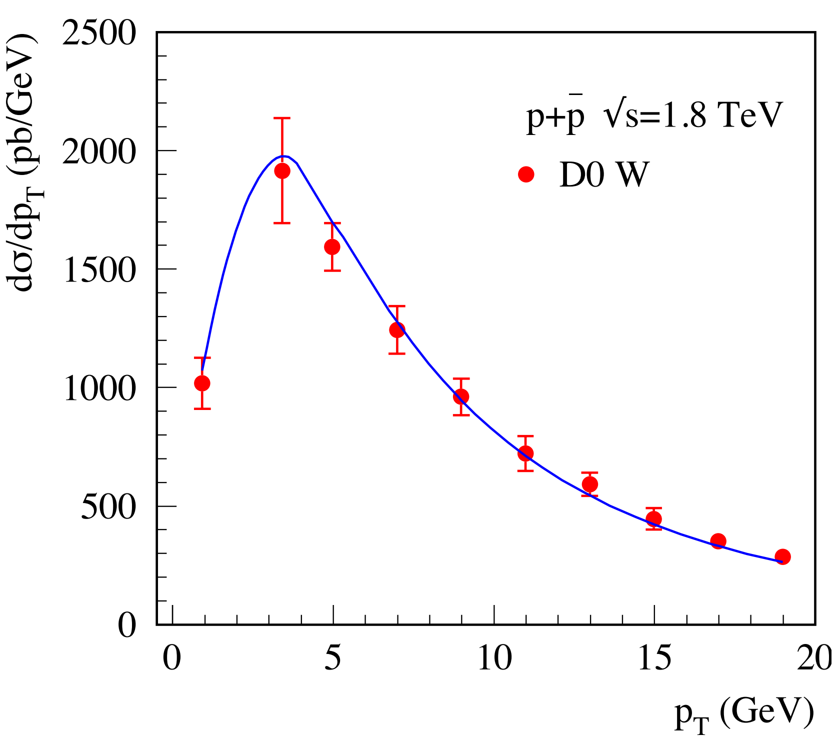
TMD evolution works

- One can figure out a way to resum the large logarithms to "all order" in your perturbation theory, and thus derive a very powerful evolution equation for the distribution



Echevarria-Idilbi-Kang-Vitev, PRD 2014

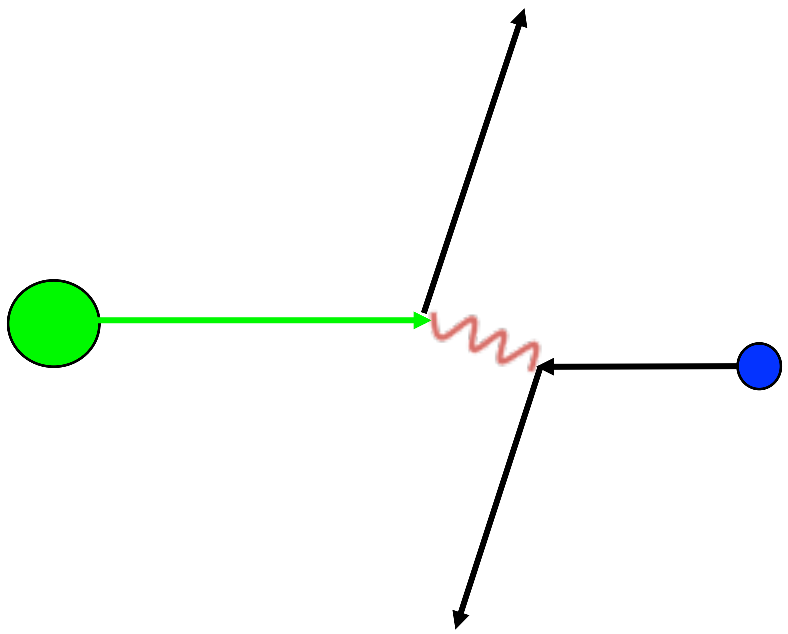


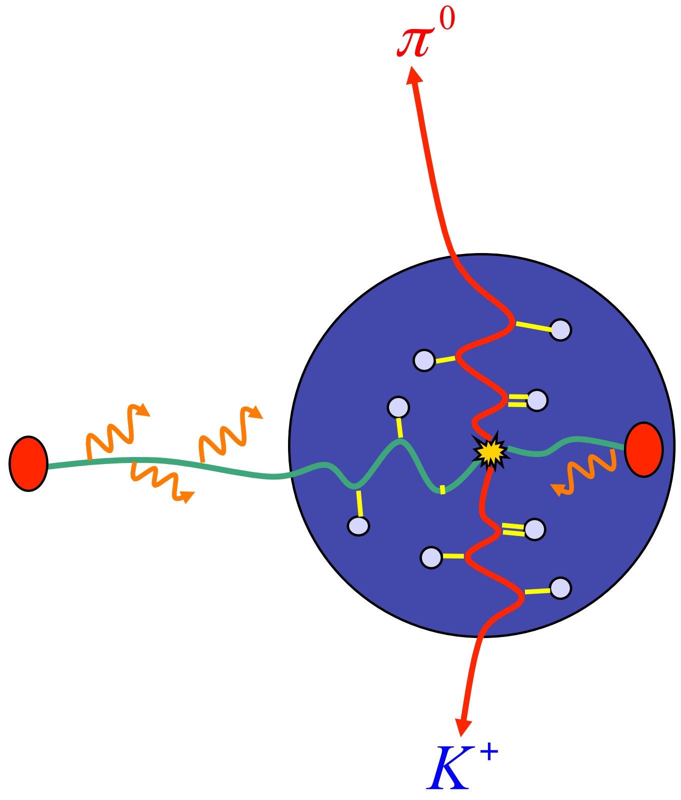


- Enough for pQCD in "vacuum"

Multiple scattering effects in nucleus collisions

- The major difference between p+p vs p+A and A+A collisions
  - p+p: usually single scattering
  - p+A, A+A: strong multiple scattering when passing through the large nucleus or the hot medium



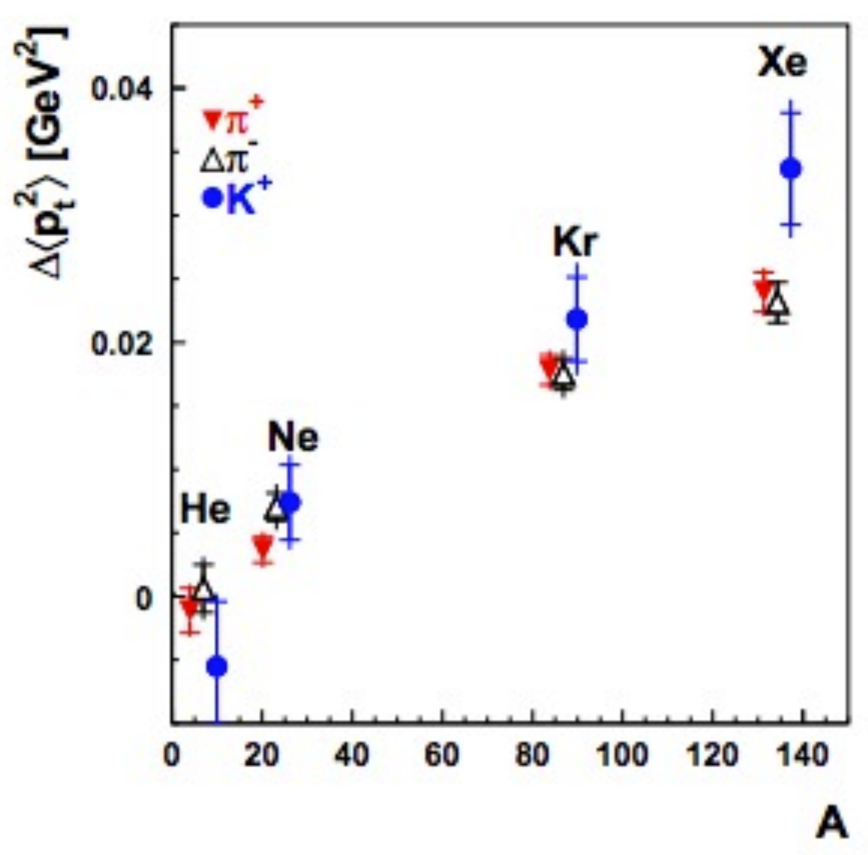


- Whether or not one might be able to describe such multiple scattering in terms of "generalized" pQCD factorization formalism

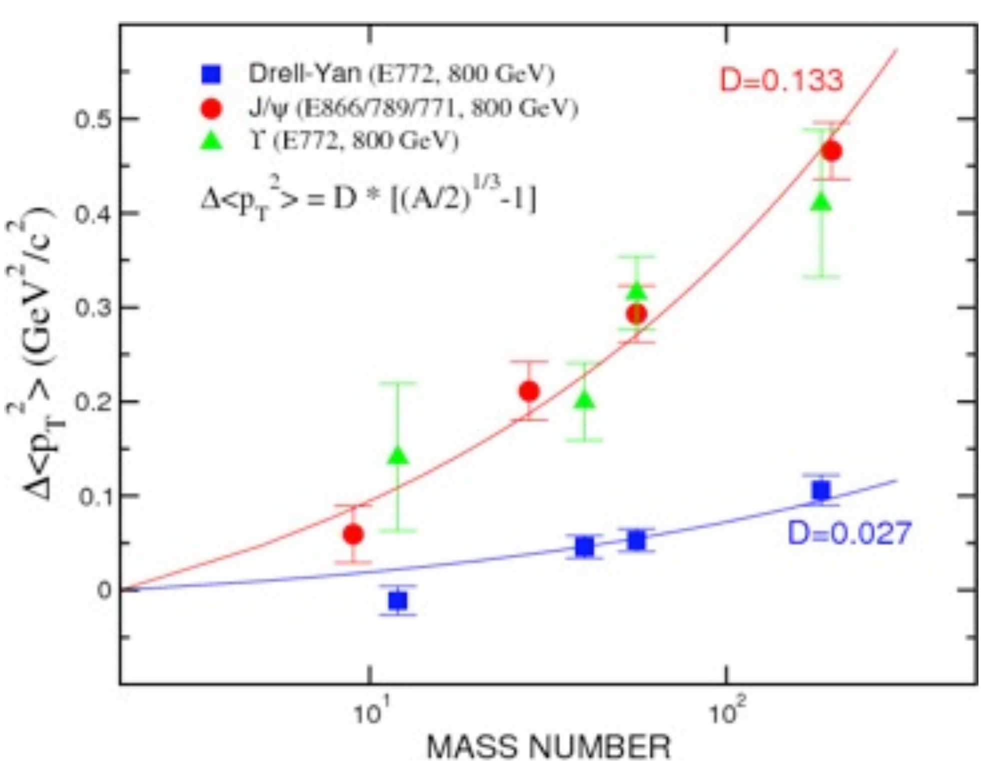
Transverse momentum broadening

- Experimental evidence of transverse momentum broadening

$$\Delta\langle q_\perp^2\rangle = \langle q_\perp^2\rangle_{pA} - \langle q_\perp^2\rangle_{pp}$$



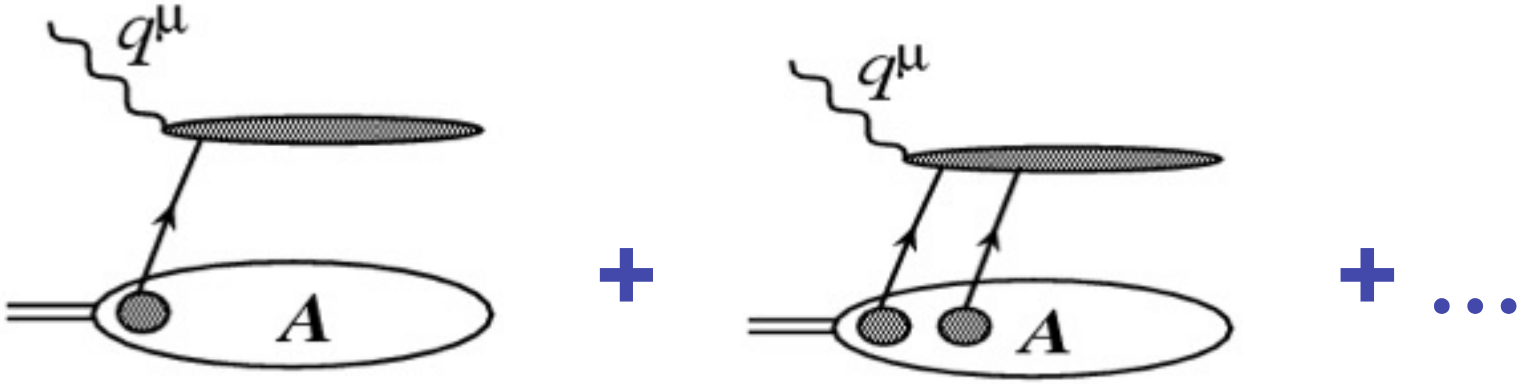
HERMES, PLB 2010



J.C. Peng 1999, M.B. Johnson et.al. , 2007

Expansion through number of scatterings

- In general the cross section can be written as an expansion of single scattering, double scattering, and even higher-order multiple scattering



$$\Delta\langle \ell_T^2\rangle = \langle \ell_T^2\rangle^{eA} - \langle \ell_T^2\rangle^{eN}$$

$$= \frac{\int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eA}}{dQ^2 d\ell_T^2}}{\frac{d\sigma_{eA}}{dQ^2}} - \frac{\int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eN}}{dQ^2 d\ell_T^2}}{\frac{d\sigma_{eN}}{dQ^2}}$$

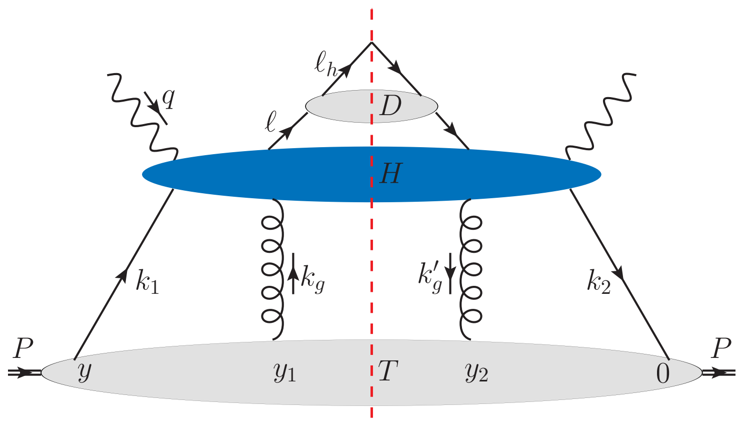
$$= \frac{\int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eA}^S}{dQ^2 d\ell_T^2} + \int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eA}^D}{dQ^2 d\ell_T^2} + \cdots}{\frac{d\sigma_{eA}}{dQ^2}} - \frac{\int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eN}^S}{dQ^2 d\ell_T^2}}{\frac{d\sigma_{eN}}{dQ^2}}$$

$$\approx \frac{\int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eA}^D}{dQ^2 d\ell_T^2}}{\frac{d\sigma_{eA}}{dQ^2}}$$

Single scattering --> localized in space
Double scattering --> leading contribution to transverse Momentum broadening

Generalized pQCD formalism: high-twist approach

- Each scattering gives a small “kt” kick to the active parton, if assuming this kick “kt << xP” the longitudinal momentum, one can perform the Taylor expansion such that the net “kt” effect can be absorbed into multi-parton correlation functions



(b)

$$k_g = x_2 P + k_\perp$$

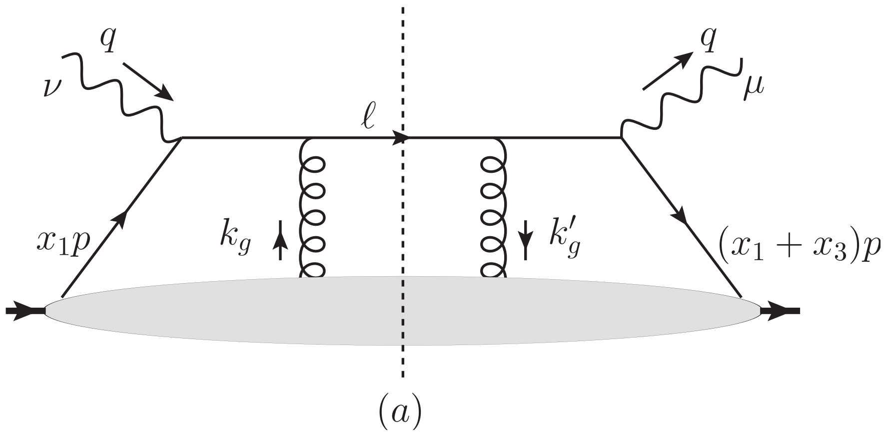
$$k'_g = (x_2 - x_3) P + k_\perp$$

Qiu-Sterman 1990, Kang-Wang-Wang-Xing, 2014, Kang-Vitev-Xing, 2013

$$\frac{dW^D_{\mu\nu}}{dz_h} = \sum_q e_q^2 \int \frac{dz}{z} D_{h/q}(z) \int \frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi}\frac{dy_2^-}{2\pi}\frac{1}{2}\langle A|\bar{\psi}_q(0)\gamma^+ F_\sigma^{\,+}(y_2^-)F^{\sigma+}(y_1^-)\psi_q(y^-)|A\rangle$$
$$\times \left[-\frac{1}{2(1-\epsilon)}g^{\alpha\beta}\right]\left[\frac{1}{2}\frac{\partial^2}{\partial k_T^\alpha \partial k_T^\beta}\overline{H}_{\mu\nu}(p,q,\ell,\ell_h,k_T)\right]_{k_T=0},$$

Leading order broadening for SIDIS

- Leading order is rather simple: depends on a new correlation function, which is universal (like PDFs/FFs). Once it is fixed from one process, it can be used to make predictions



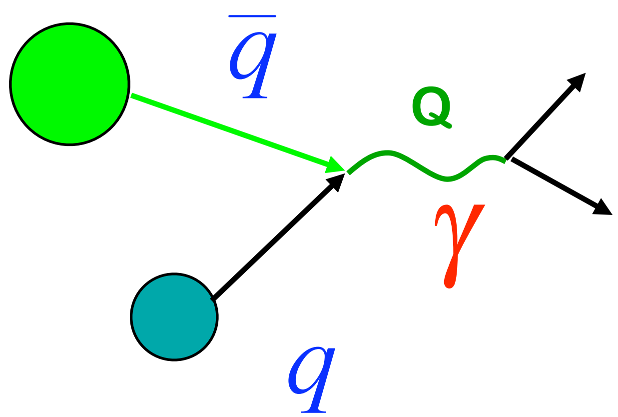
Guo 1998, Guo-Qiu 2000

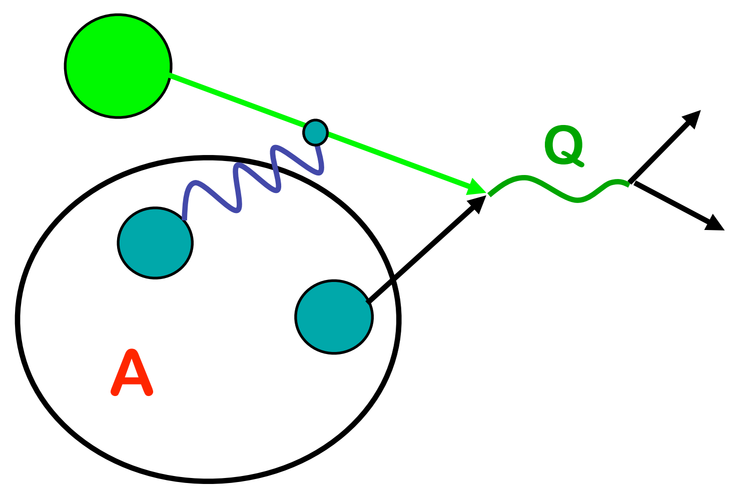
$$\Delta\langle \ell_{hT}^2\rangle = \left(\frac{4\pi^2\alpha_s z_h^2}{N_c}\right)\frac{\sum_q e_q^2 T_{qg}(x_B,0,0) D_{h/q}(z_h)}{\sum_q e_q^2 f_{q/A}(x_B) D_{h/q}(z_h)}$$

Similar for Drell-Yan production

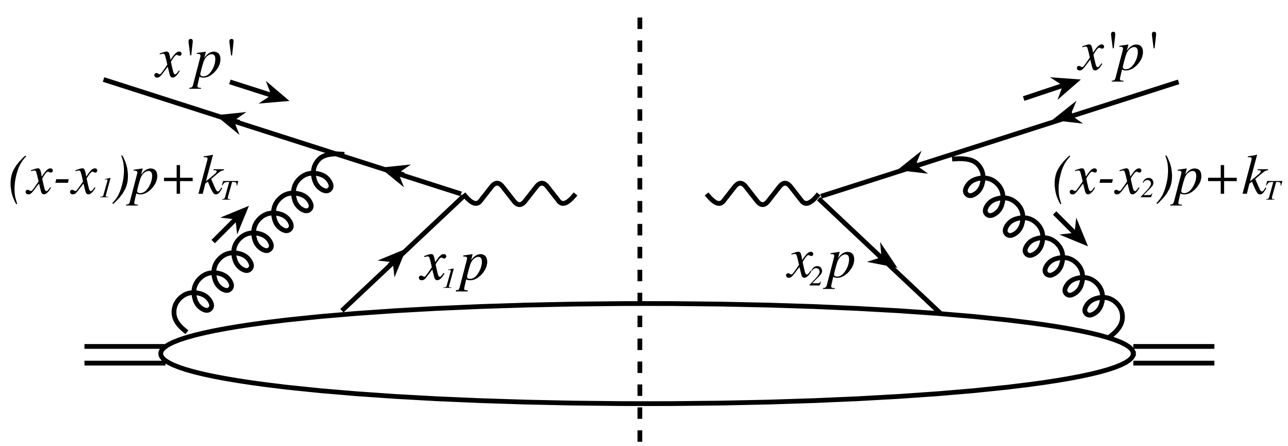
- Single and double scattering

Kang-Qiu, 2008





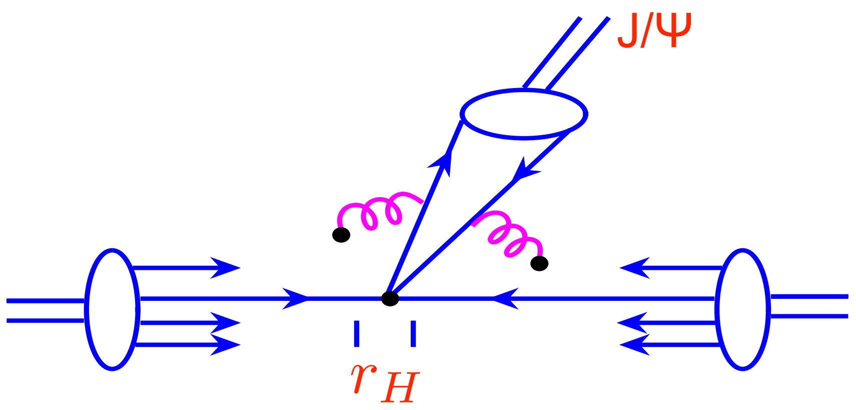
- LO calculation in p+A collisions



$$\langle q_T^2\rangle_{pA} = \left(\frac{8\pi^2\alpha_s}{N_c^2-1}\,C_F\right)\frac{\sum_q\int dx'\,\phi_{\bar{q}/h}(x')\int dx\,T_{qg}(x,0,0)\,\frac{d\hat{\sigma}_{q\bar{q}}}{dQ^2}}{\sum_q\int dx'\,\phi_{\bar{q}/h}(x')\int dx\,\phi_{q/A}(x)\,\frac{d\hat{\sigma}_{q\bar{q}}}{dQ^2}}$$

Generalized to J/ψ production

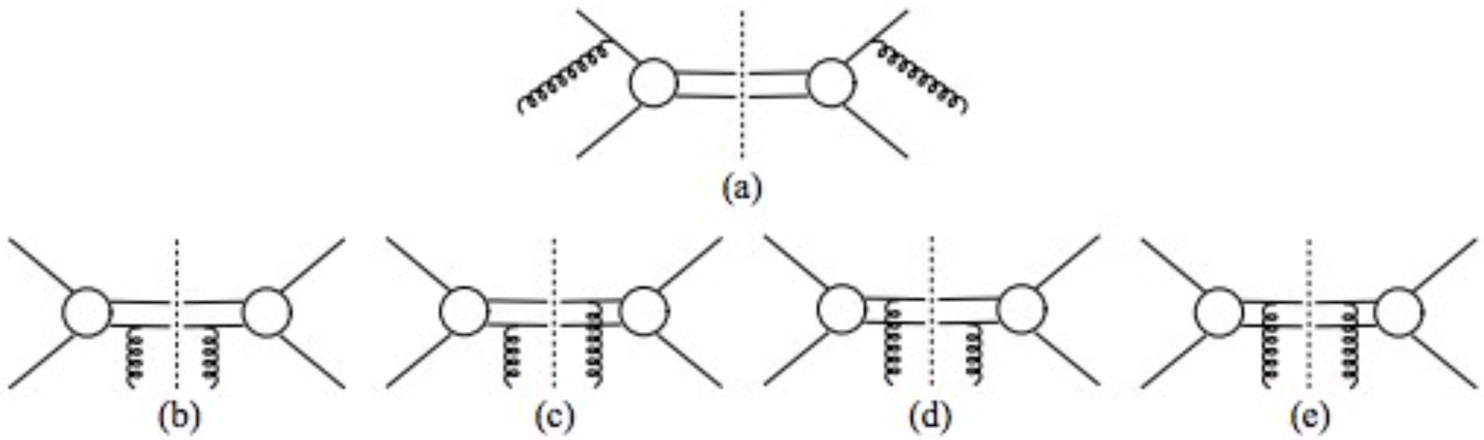
- There could be both initial-state and final-state multiple scattering to J/ψ production



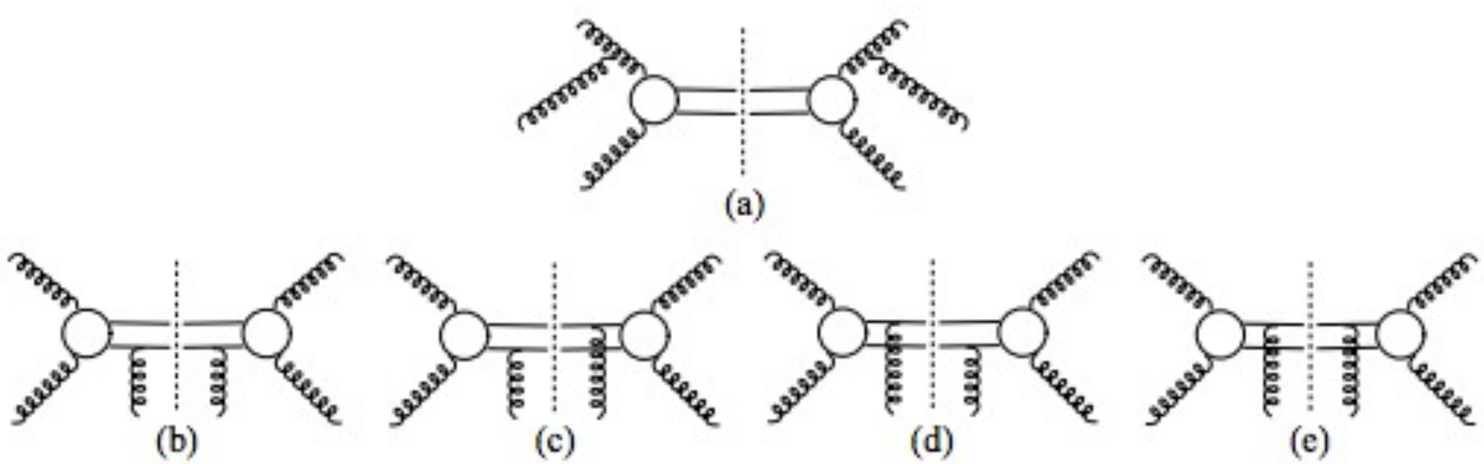
- Multiple scattering on the c-cbar pair could change the pair's momentum as well as color
  - Net effect on broadening depends on how quarkonium is formed
  - Color Evaporation Model (CEM): sensitive to the change of the momentum but not the color
  - NRQCD: sensitive to both meomentum and the color (related to different perturbative NRQCD matrix elements)

Initial and final state interaction

- Quark-antiquark annihilation channel



- Gluon-gluon fusion channel



Transverse momentum broadening for quarkonium

- Color evaporation model

$$\Delta\langle q_T^2\rangle_{\rm HQ}^{\rm CEM} = \left(\frac{8\pi^2\alpha_s}{N_c^2-1}\,\lambda^2\,A^{1/3}\right)\frac{(C_F+C_A)\,\sigma_{q\bar{q}}+2\,C_A\,\sigma_{gg}}{\sigma_{q\bar{q}}+\sigma_{gg}}$$

- NRQCD model

$$\Delta\langle q_T^2\rangle_{\rm HQ}^{\rm NRQCD} = \left(\frac{8\pi^2\alpha_s}{N_c^2-1}\,\lambda^2\,A^{1/3}\right)\frac{(C_F+C_A)\,\sigma_{q\bar{q}}^{(0)}+2C_A\,\sigma_{gg}^{(0)}+\sigma_{q\bar{q}}^{(1)}}{\sigma_{q\bar{q}}^{(0)}+\sigma_{gg}^{(0)}}$$

- different analytic expression
  but very similar numerical results

Kang-Qiu, 2008, 2012

Transverse momentum broadening for quarkonium

- Color evaporation model

$$\Delta\langle q_T^2\rangle_{\rm HQ}^{\rm CEM} = \left(\frac{8\pi^2\alpha_s}{N_c^2-1}\,\lambda^2\,A^{1/3}\right)\frac{(C_F+C_A)\,\sigma_{q\bar{q}} + 2\,C_A\,\sigma_{gg}}{\sigma_{q\bar{q}}+\sigma_{gg}}$$

- NRQCD model

$$\Delta\langle q_T^2\rangle_{\rm HQ}^{\rm NRQCD} = \left(\frac{8\pi^2\alpha_s}{N_c^2-1}\,\lambda^2\,A^{1/3}\right)\frac{(C_F+C_A)\,\sigma_{q\bar{q}}^{(0)} + 2C_A\,\sigma_{gg}^{(0)} + \sigma_{q\bar{q}}^{(1)}}{\sigma_{q\bar{q}}^{(0)}+\sigma_{gg}^{(0)}}$$

- different analytic expression
  but very similar numerical results

$$\Delta\langle p_T^2\rangle_{J/\psi,\Upsilon} \sim 2\frac{C_A}{C_F}\Delta\langle p_T^2\rangle_{DY}$$

Kang-Qiu, 2008, 2012

Transverse momentum broadening for quarkonium

- Color evaporation model

$$\Delta\langle q_T^2\rangle_{\rm HQ}^{\rm CEM} = \left(\frac{8\pi^2\alpha_s}{N_c^2-1}\,\lambda^2\,A^{1/3}\right)\frac{(C_F+C_A)\,\sigma_{q\bar q}+2\,C_A\,\sigma_{gg}}{\sigma_{q\bar q}+\sigma_{gg}}$$

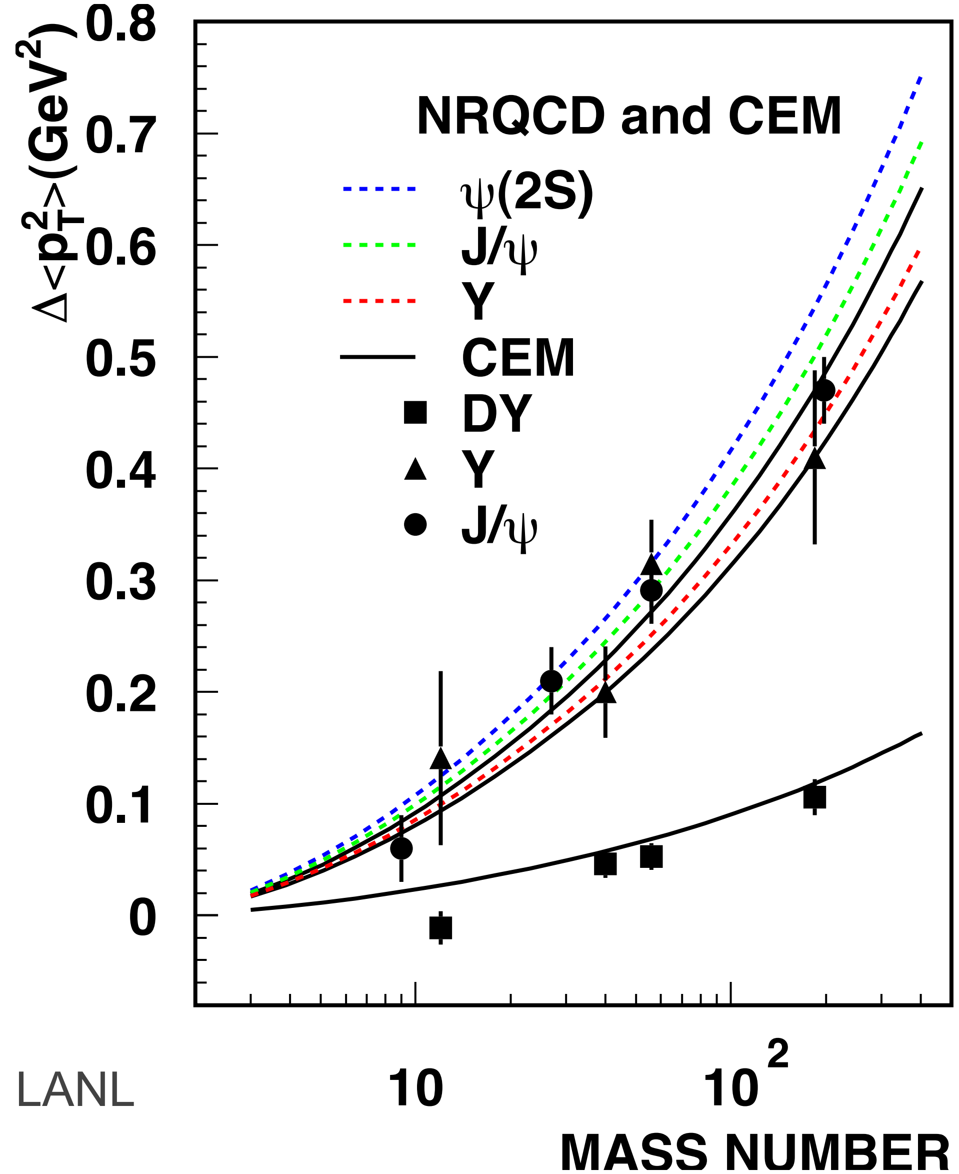
- NRQCD model

$$\Delta\langle q_T^2\rangle_{\rm HQ}^{\rm NRQCD} = \left(\frac{8\pi^2\alpha_s}{N_c^2-1}\,\lambda^2\,A^{1/3}\right)\frac{(C_F+C_A)\,\sigma_{q\bar q}^{(0)}+2C_A\,\sigma_{gg}^{(0)}+\sigma_{q\bar q}^{(1)}}{\sigma_{q\bar q}^{(0)}+\sigma_{gg}^{(0)}}$$

- different analytic expression
  but very similar numerical results

$$\Delta\langle p_T^2\rangle_{J/\psi,\Upsilon} \sim 2\frac{C_A}{C_F}\Delta\langle p_T^2\rangle_{DY}$$

Kang-Qiu, 2008, 2012

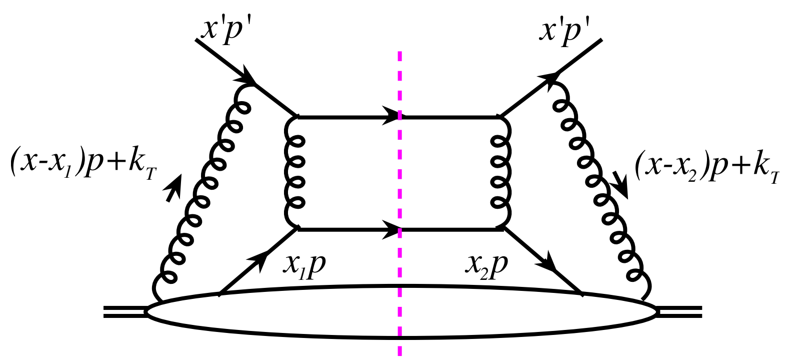


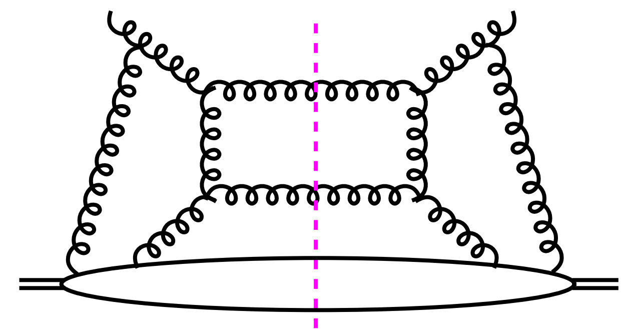
Dijet production in pA collisions

- Both initial- and final-state interactions contribute: many more processes, the calculation is tedious

Kang-Vitev-Xing, PRD 2012

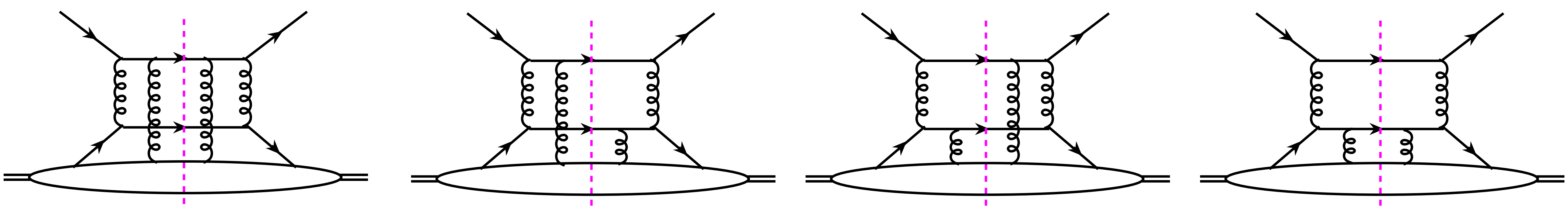
  - initial-state





(c)

  - final-state



Final result

- Nuclear broadening for dijet momentum imbalance in p+A collisions

$$\Delta\langle q_\perp^2\rangle_{pA} = \left(\frac{8\pi^2\alpha_s}{N_c^2-1}\right)\frac{\sum_{a,b}\frac{f_{a/p}(x')}{x'x}\left[T_{b/A}^{(I)}(x)H_{ab\to cd}^{I}(\hat{s},\hat{t},\hat{u})+T_{b/A}^{(F)}(x)H_{ab\to cd}^{F}(\hat{s},\hat{t},\hat{u})\right]}{\sum_{a,b}\frac{f_{a/p}(x')f_{b/p}(x)}{x'x}H_{ab\to cd}^{U}(\hat{s},\hat{t},\hat{u})}$$

- Result for dihadron:

$$\Delta\langle q_\perp^2\rangle = \left(\frac{8\pi^2\alpha_s}{N_c^2-1}\right)\frac{\sum_{abcd}\int\frac{dz_1}{z_1}D_{h_1/c}(z_1)D_{h_2/d}(z_2)\frac{f_{a/p}(x')}{x'x}\left[T_{b/A}^{(I)}(x)H_{ab\to cd}^{I}(\hat{s},\hat{t},\hat{u})+T_{b/A}^{(F)}(x)H_{ab\to cd}^{F}(\hat{s},\hat{t},\hat{u})\right]}{\sum_{abcd}\int\frac{dz_1}{z_1}D_{h_1/c}(z_1)D_{h_2/d}(z_2)\frac{f_{a/p}(x')f_{b/p}(x)}{x'x}H_{ab\to cd}^{U}(\hat{s},\hat{t},\hat{u})}$$

- The strength of broadening depends on the color
  - initial state multiple scattering

$$H_{ab\to cd}^{I} = \begin{cases} C_F H_{ab\to cd}^{U} & \text{a=quark} \\ C_A H_{ab\to cd}^{U} & \text{a=gluon} \end{cases},$$

Final-state multiple scattering

- Again the strength of the broadening depends on the color configuration

$$
\begin{aligned}
H^F_{qq'\to qq'} &= \frac{(N_c^2-3)(N_c^2-1)}{2N_c^3}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right], \\
H^F_{qq\to qq} &= \frac{(N_c^2-3)(N_c^2-1)}{2N_c^3}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right]+\frac{2(N_c^2-1)}{N_c^4}\left[\frac{\hat{s}^2}{\hat{t}\hat{u}}\right], \\
H^F_{q\bar{q}\to q'\bar{q}'} &= C_A H^U_{q\bar{q}\to q'\bar{q}'}, \\
H^F_{q\bar{q}\to q\bar{q}} &= C_A H^U_{q\bar{q}\to q\bar{q}} - \frac{(N_c^2-1)^2}{2N_c^3}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right], \\
H^F_{qg\to qg} &= C_F H^U_{qg\to qg} - \frac{N_c}{2}\left[\frac{\hat{s}(\hat{s}^2+\hat{u}^2)}{\hat{t}^2\hat{u}}\right], \\
H^F_{q\bar{q}\to gg} &= C_A H^U_{q\bar{q}\to gg} - \frac{N_c^2-1}{2N_c^2}\left[\frac{\hat{t}}{\hat{u}}+\frac{\hat{u}}{\hat{t}}\right], \\
H^F_{gg\to q\bar{q}} &= C_A H^U_{gg\to q\bar{q}} - \frac{1}{2(N_c^2-1)}\left[\frac{\hat{t}}{\hat{u}}+\frac{\hat{u}}{\hat{t}}\right], \\
H^F_{gg\to gg} &= C_A H^U_{gg\to gg} + \frac{2N_c^3}{(N_c^2-1)^2}\left[\frac{\hat{t}}{\hat{u}}+\frac{\hat{u}}{\hat{t}}+1\right]^2
\end{aligned}
$$

The width of the away-side peak

- One could derive the relation between the width of the away-side peak and the transverse momentum broadening

$$\cos^2\left(\sqrt{\frac{2}{\pi}}\sigma_F\right) = \frac{1 - \frac{\langle|j_{\perp y}|\rangle^2}{p^2_{\perp,\mathrm{assoc}}}}{1 + \frac{\langle|j_{\perp y}|\rangle^2}{p^2_{\perp,\mathrm{trig}}} + \frac{1}{\pi}\frac{\langle q_\perp^2\rangle}{p^2_{\perp,\mathrm{trig}}}}$$

- Dashed: the result without broadening

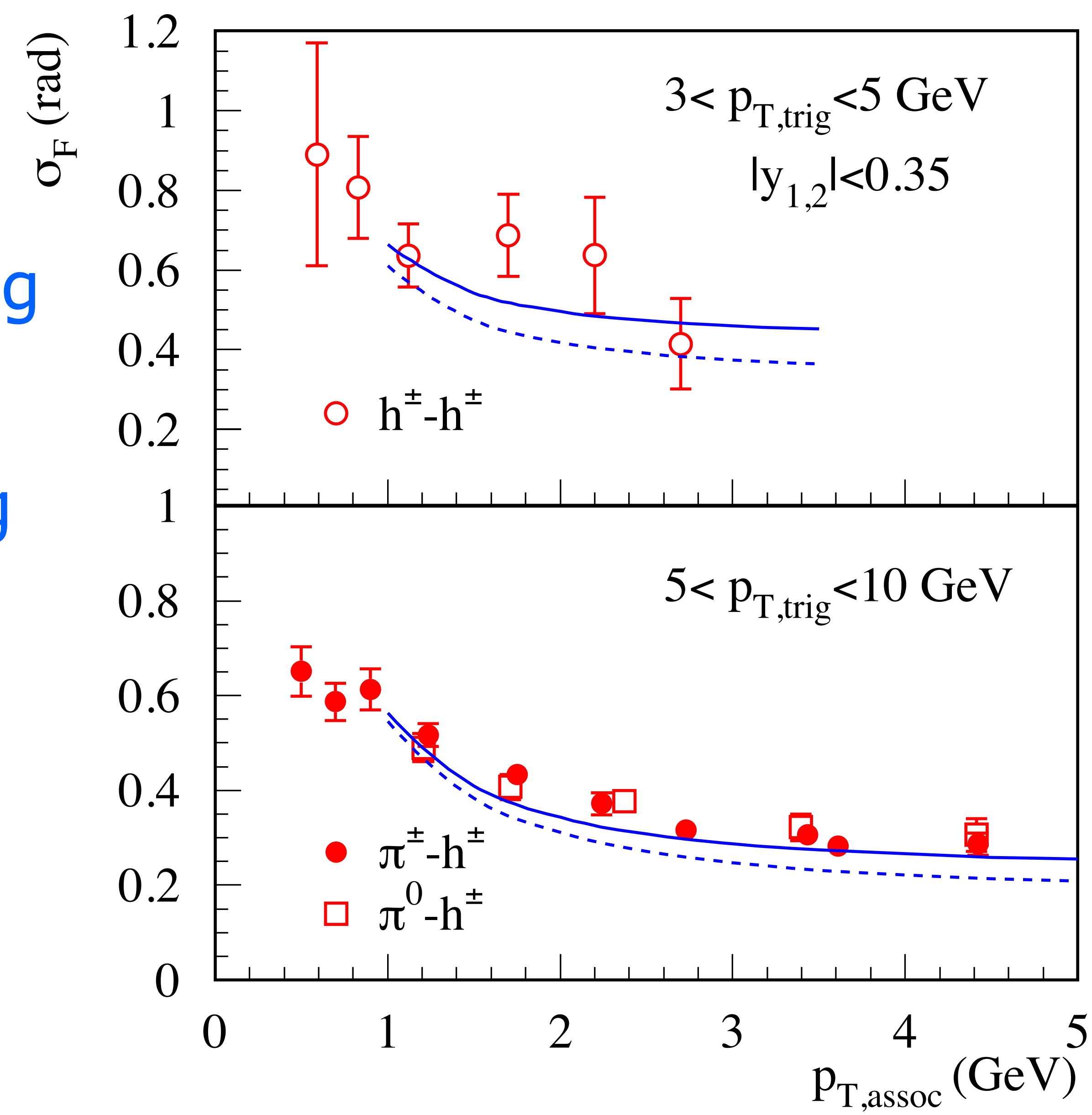
- Solid: the calculation with broadening

$$\Delta\langle q_\perp^2\rangle_{\mathrm{pA}} \propto \langle F^{+\alpha}F_\alpha{}^{+}\rangle A^{1/3}$$

Local gluon density

Length

Kang-Vitev-Xing, PRD, 2012

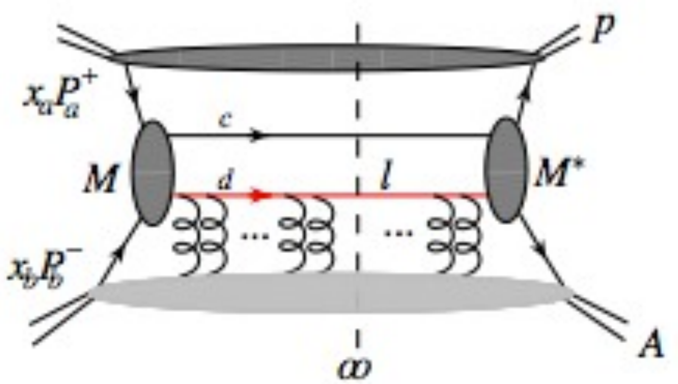


PHENIX collaboration, PRC, 2006

Multiple scattering could also modify the production rate

- These coherent multiple scattering also modify the differential cross sections: forward direction $R_{pA} = \frac{\sigma_{pA}}{A\,\sigma_{pp}}$

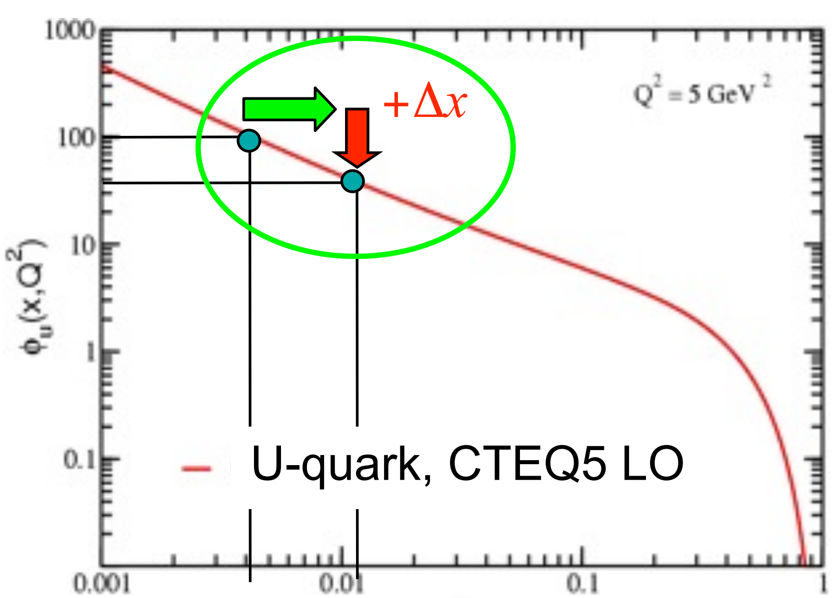
Kang-Vitev-Xing, PRD, 2012

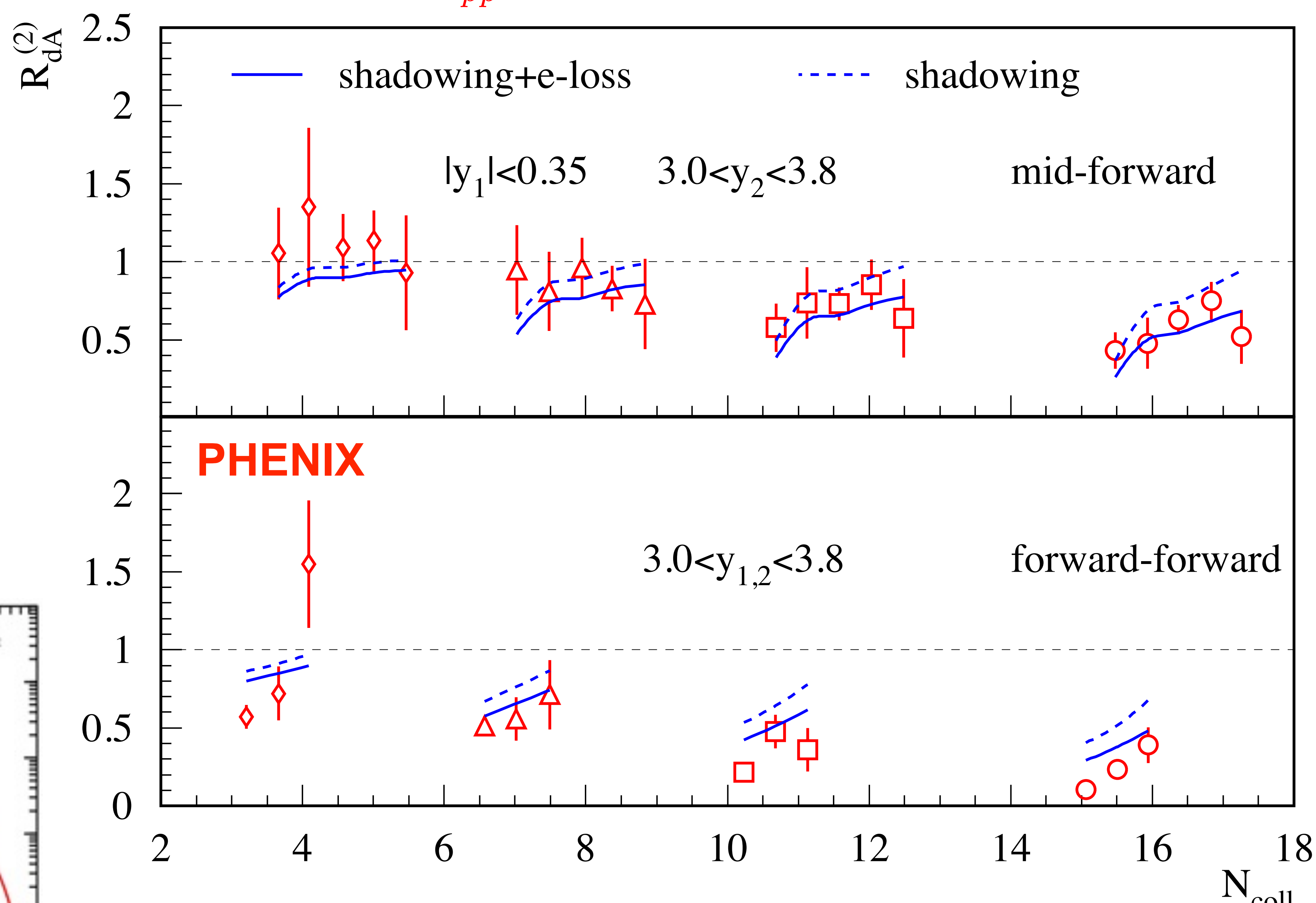


final parton is dressed-up push the shift in x

$$x \to x\left(1 + \frac{\xi^2}{-\hat{t}} A^{1/3}\right)$$

$$\xi^2 \propto \langle F^{+\alpha} F_\alpha{}^{+} \rangle$$



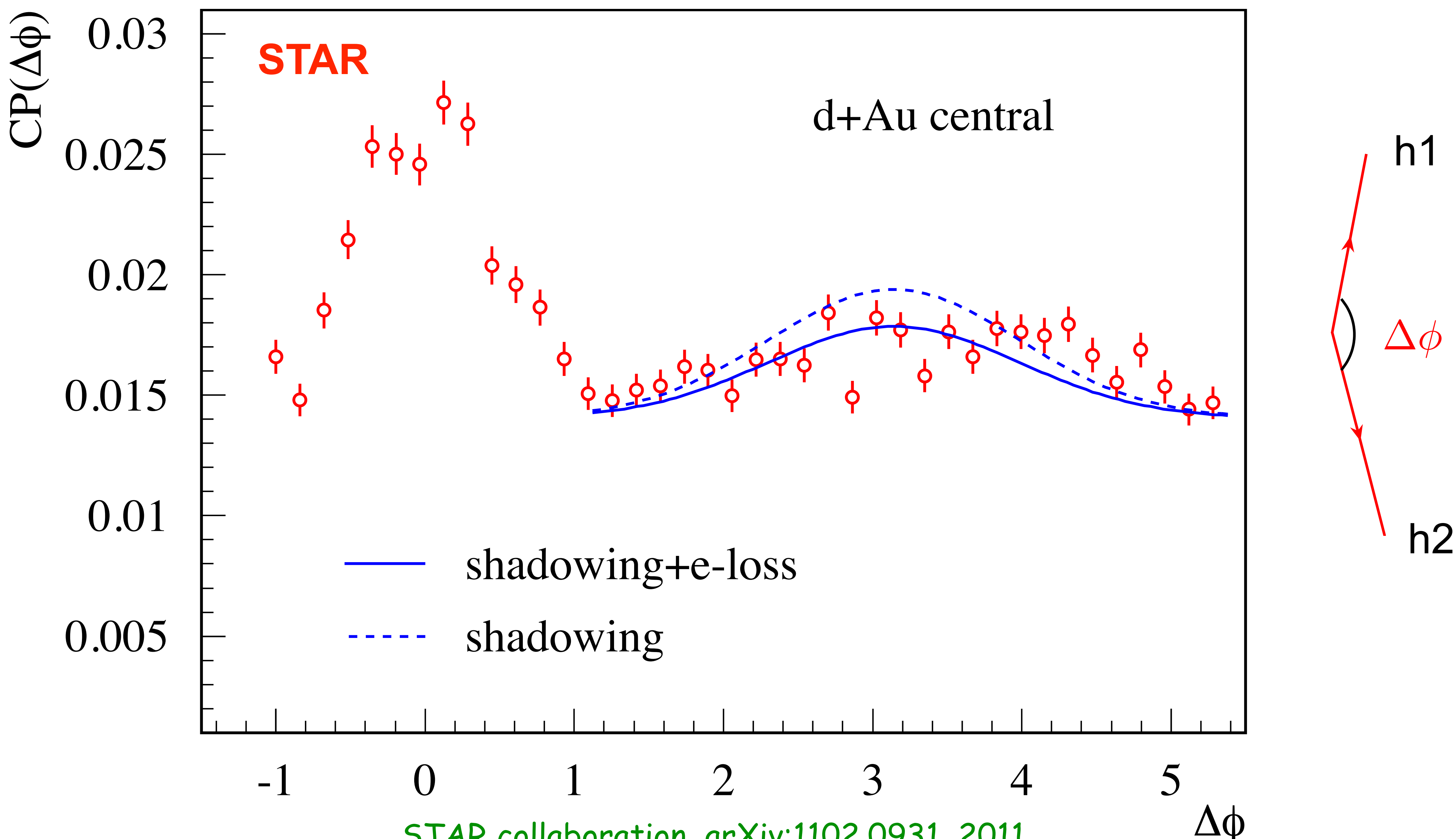


PHENIX collaboration, PRL, 2011

The azimuthal distribution

- Combine the modification on the away-side width and the cross section, we calculated the away-side peak, which describes the STAR data perfectly

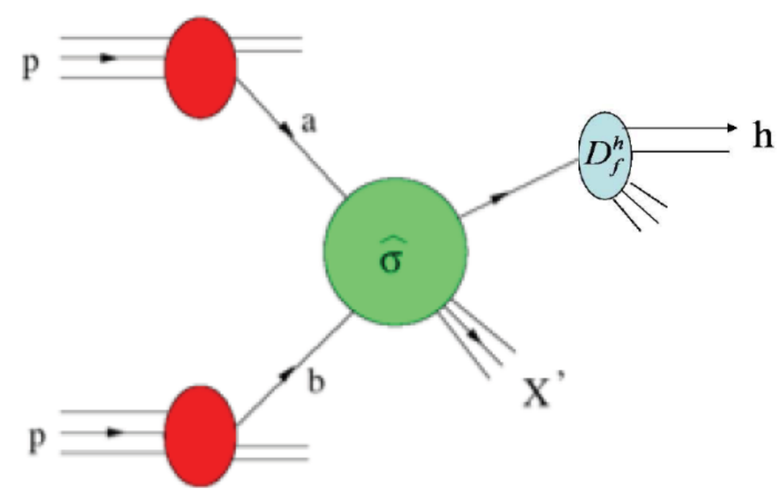
Kang-Vitev-Xing, PRD, 2012



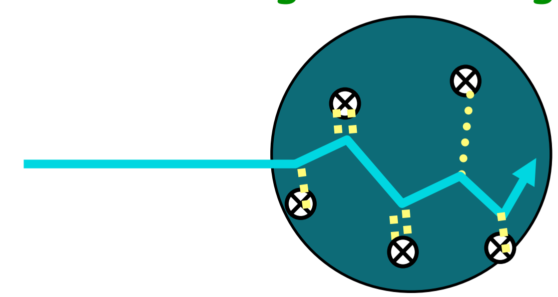
STAR collaboration, arXiv:1102.0931, 2011

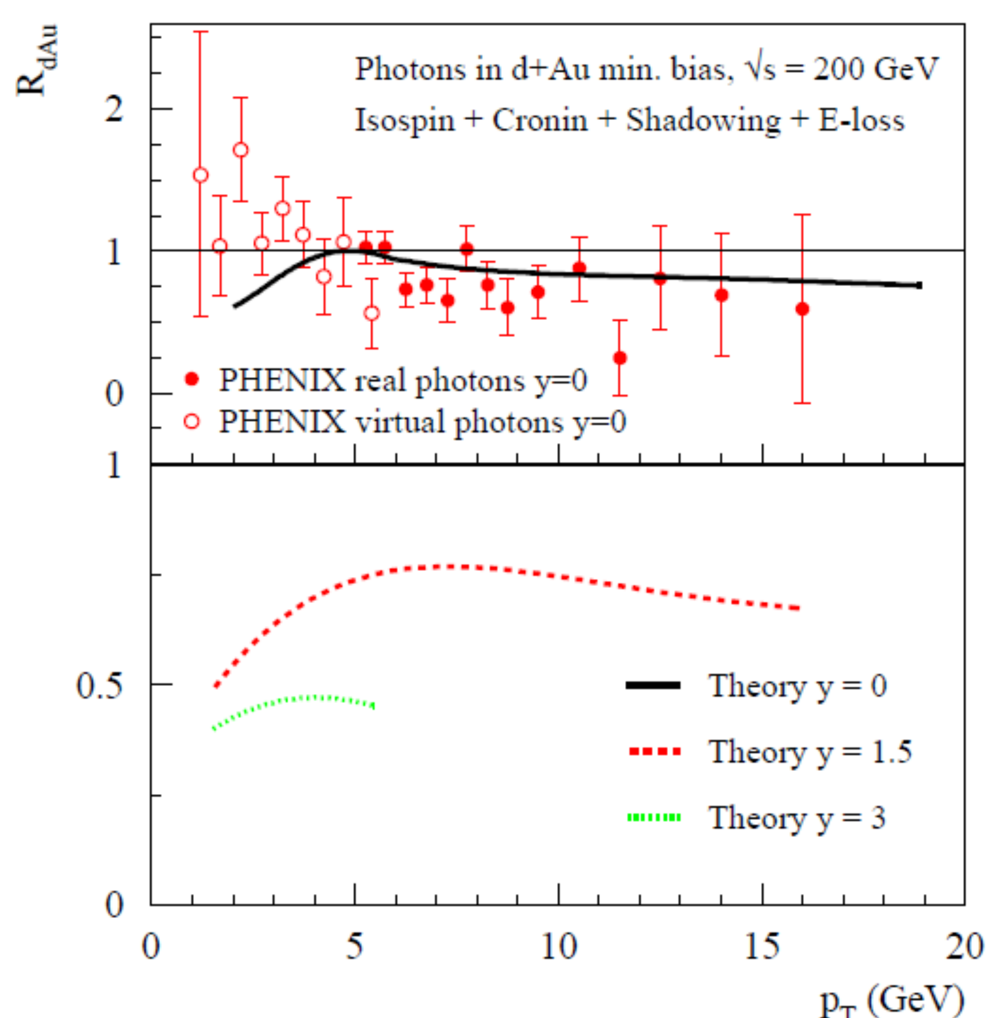
For single inclusive hadron production: p+A→h+X

- Works very well for single inclusive hadron production

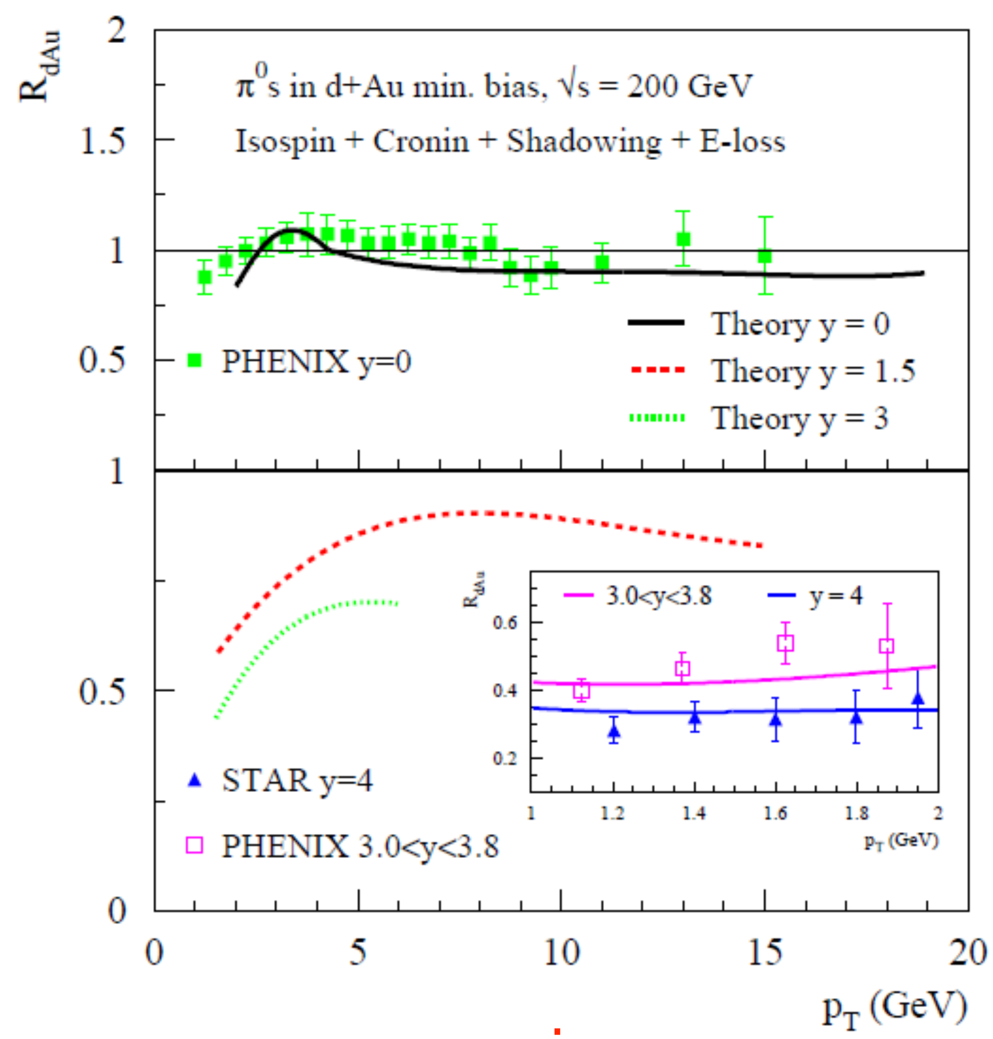


Kang-Vitev-Xing, PLB, 2012





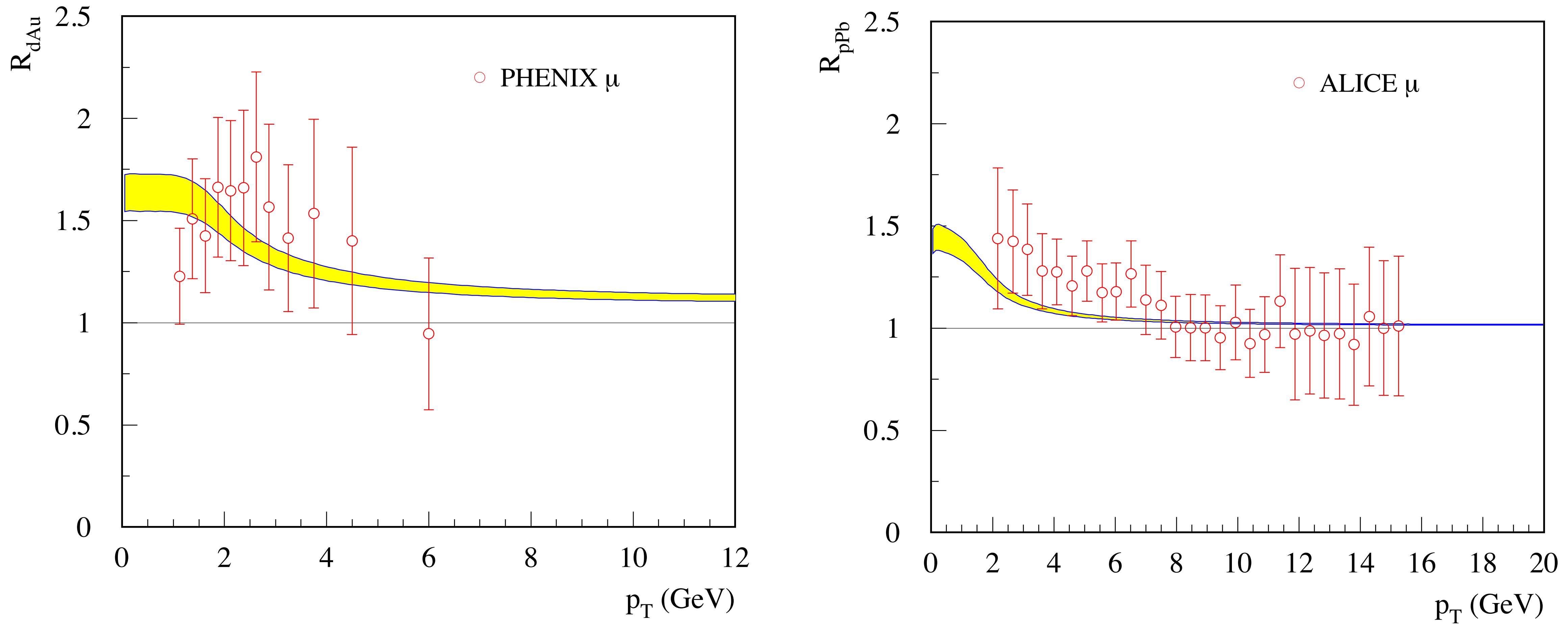
photon



pion

Backward rapidity region (large x region)

- In the backward rapidity region, the parton momentum fraction x (in the nucleus) is relatively large. Incoherent vs coherent multiple scattering



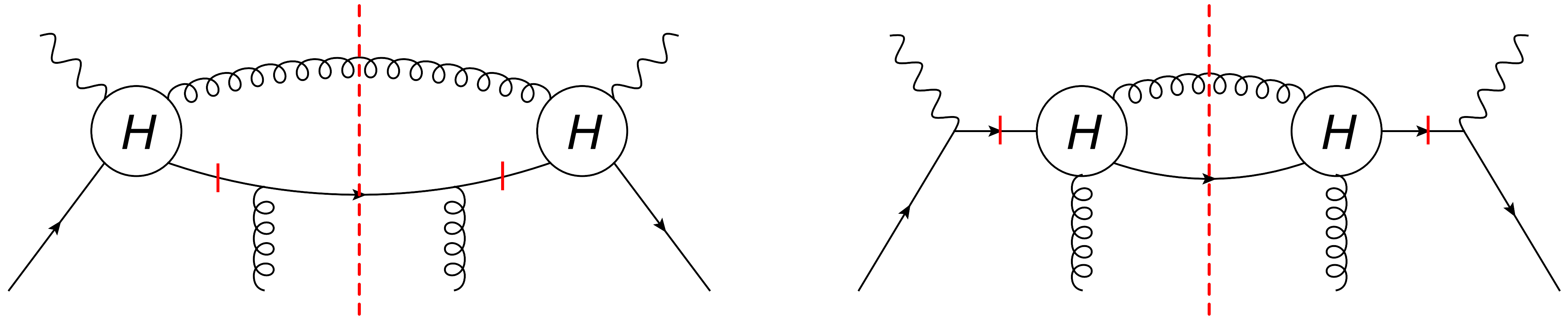
Kang-Vitev-Xing, PRD 2013

Kang-Vitev-Xing, 2014, to appear

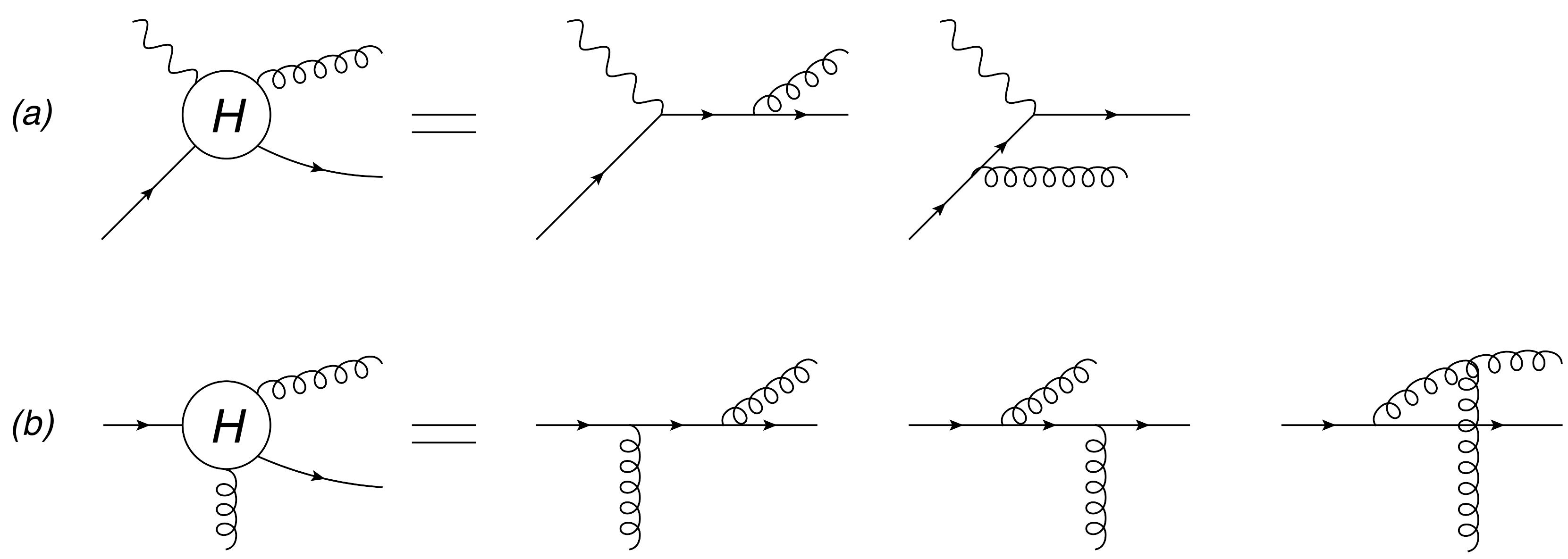
QCD factorization beyond LO

- So far all the calculations are based on LO calculation. To ensure QCD factorization formalism and to derive evolution equation for the relevant correlation functions, we have to go beyond LO
  - SIDIS as an example to illustrate: real diagram

Kang-Wang-Wang-Xing, PRL 2014

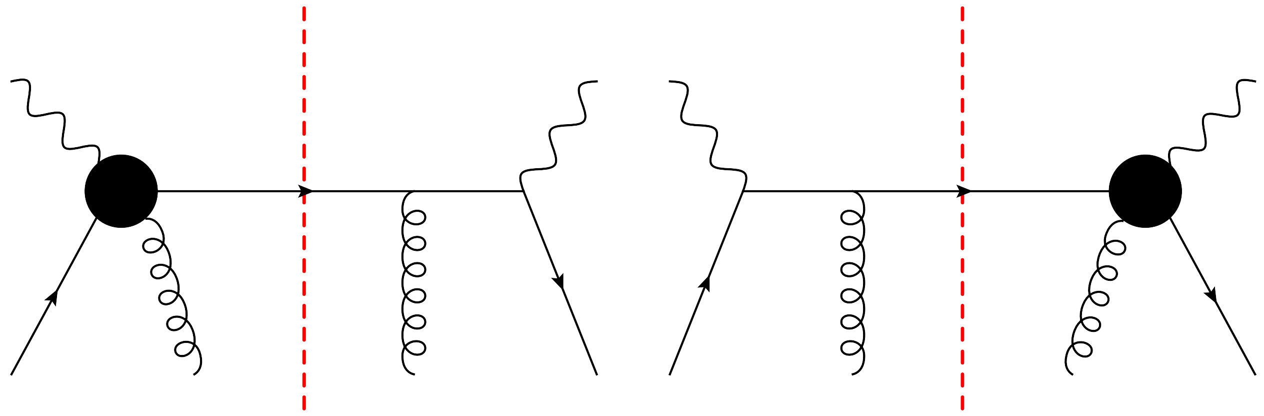


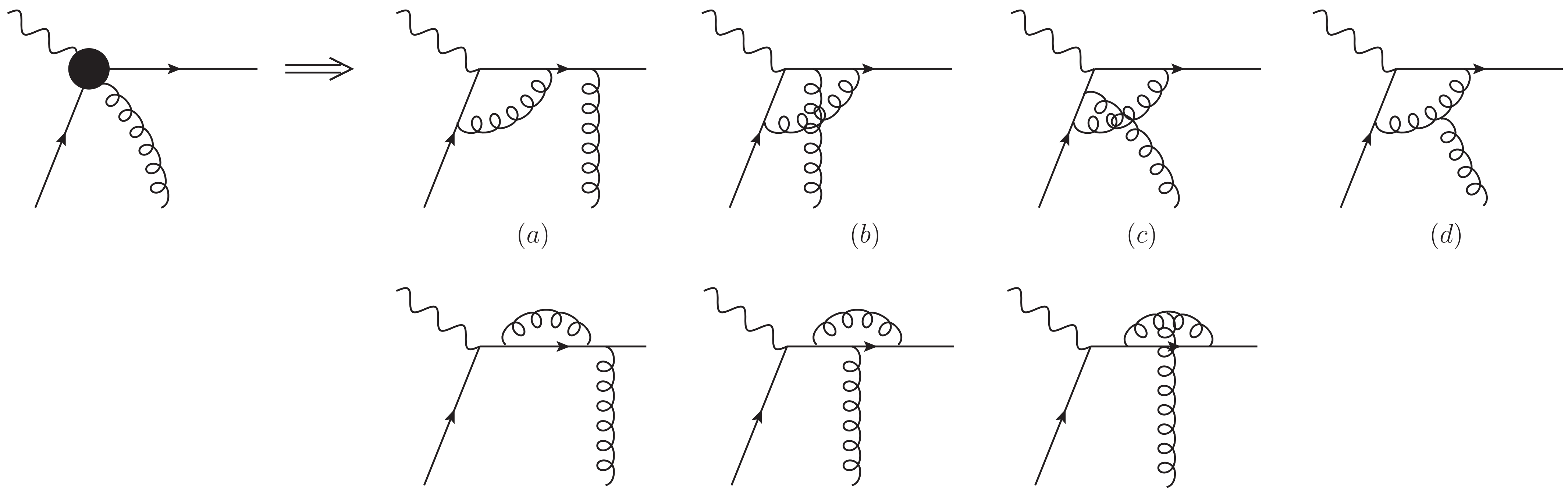
(a) (b)



Virtual diagrams for SIDIS

- Virtual diagrams are much more complicated than the usual vertex correction encountered in standard QFT textbook





QCD evolution equation

- Final result: again collinear divergence

Kang-Wang-Wang-Xing, PRL 2014

$$\frac{d\langle \ell_{hT}^2 \sigma^D\rangle}{d\mathcal{PS}} = \sigma_h \frac{\alpha_s}{2\pi} \sum_q e_q^2 \int \frac{dz}{z} D_{h/q}(z) \int \frac{dx}{x} \Bigg\{ \left(-\frac{1}{\hat\epsilon} + \ln\frac{Q^2}{\mu^2}\right) \Big[\delta(1-\hat x) P_{qq}(\hat z) T_{qg}(x,0,0)$$
$$+ \delta(1-\hat z)\Big(\mathcal{P}_{qg\to qg}\otimes T_{qg} + P_{qg}(\hat x) T_{gg}(x,0,0)\Big)\Big]$$
$$+ H_{qg}^{C-R}\otimes T_{qg} + H_{qg}^{C-V}\otimes T_{qg} - H_{qg}^{A}\otimes T_{qg}^{A} + H_{gg}^{C}\otimes T_{gg} \Bigg\}$$

- Reabsorb collinear divergence back to "renormalized" correlation function

$$T_{qg}(x_B,0,0,\mu_f^2) = T_{qg}(x_B,0,0) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat\epsilon} + \ln\frac{\mu^2}{\mu_f^2}\right) \int_{x_B}^1 \frac{dx}{x}\Big[\mathcal{P}_{qg\to qg}\otimes T_{qg} + P_{qg}(\hat x) T_{gg}(x,0,0)\Big]$$

- DGLAP evolution equation

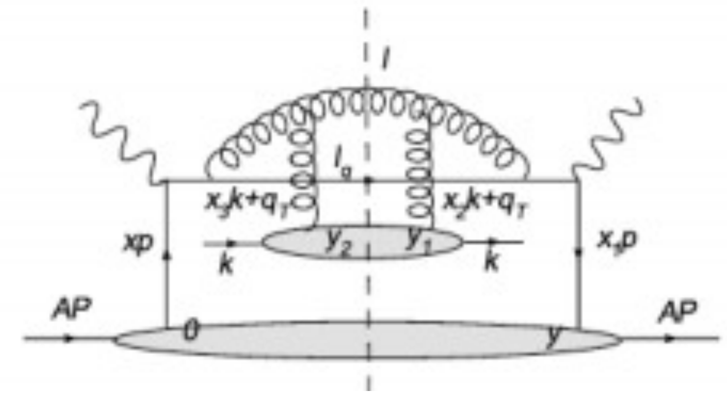
$$\mu_f^2 \frac{\partial}{\partial \mu_f^2} T_{qg}(x_B,0,0,\mu_f^2) = \frac{\alpha_s}{2\pi}\int_{x_B}^1 \frac{dx}{x}\Big[\mathcal{P}_{qg\to qg}\otimes T_{qg} + P_{qg}(\hat x) T_{gg}(x,0,0,\mu_f^2)\Big]$$

Connection to Jet transport coefficient

- Under some assumption, one can relate the correlation function to the qhat in this formalism

$$T_{qg}(x_B, 0, 0, \mu_f^2) \approx \frac{N_c}{4\pi^2\alpha_s} f_{q/A}(x_B, \mu_f^2) \int dy^- \hat{q}(\mu_f^2, y^-)$$

J. Casalderrey-Solana and X.N. Wang, 2008



- In the large x region

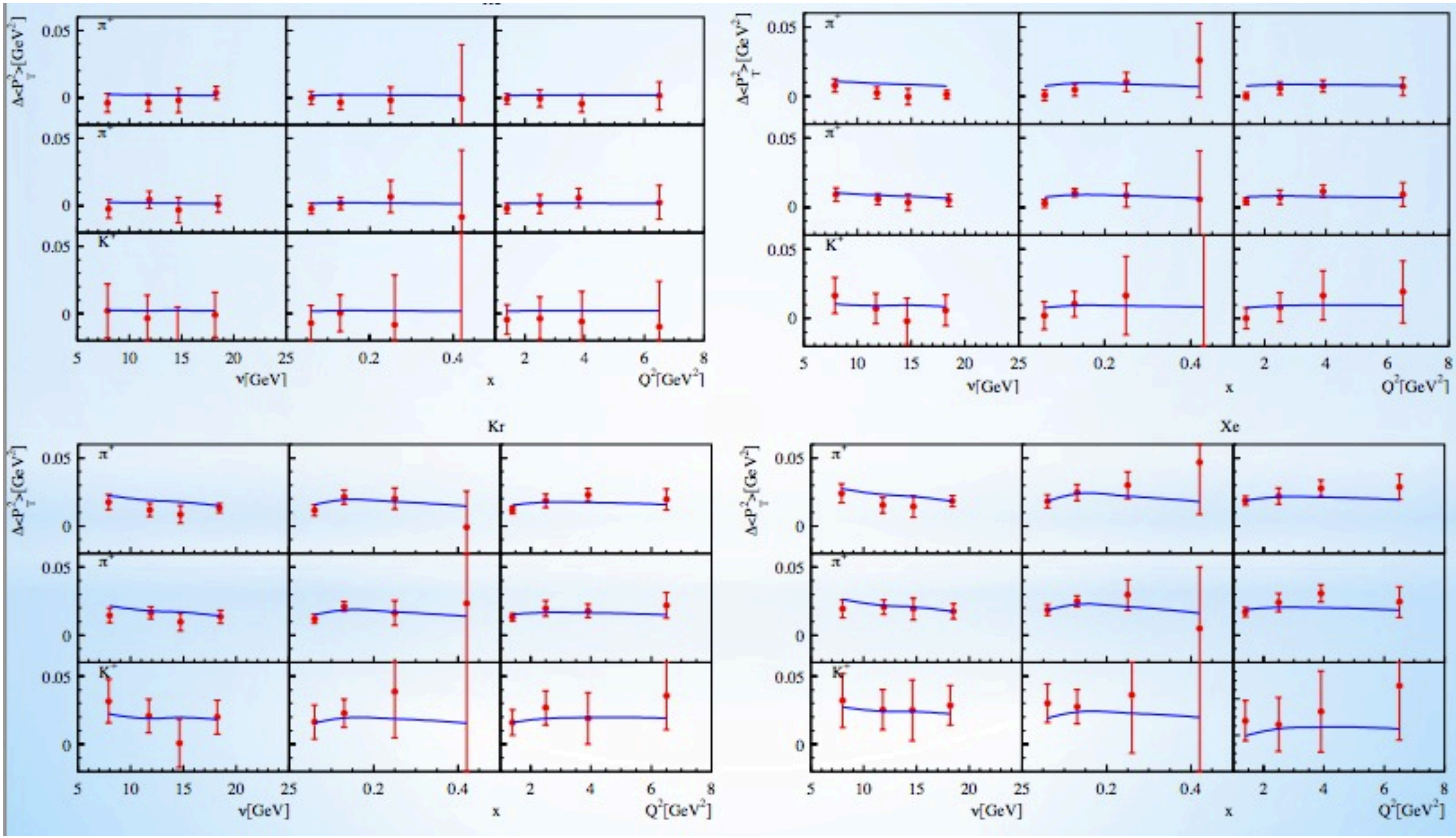
$$\mu^2 \frac{\partial \hat{q}}{\partial \mu^2} = 0$$

- In the intermediate x region

$$\mu^2 \frac{\partial \hat{q}(\mu^2)}{\partial \mu^2} = \frac{\alpha_s}{2\pi} C_A \ln(1/x_B) \hat{q}(\mu^2)$$

Describe the HERMES data

Kang-Wang-Wang-Xing, to appear

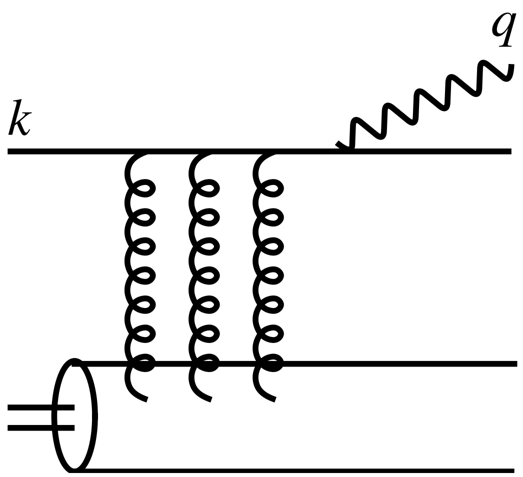


Recover full kt information

- So far we assume "kt" kick is relatively small, we thus perform kt expansion and include the net kt information in the multi-parton correlation function
- What if the kt is very important (comparable or dominant, e.g., in the small-x limit), then one has to keep the full kt dependence in the calculation (resum multiple scattering to all order)
  - Small-x formalism is one of such formalisms which resum the parton multiple scattering to all order (into Wilson line or gauge link)

Drell-Yan production in small-x regime

- At leading order, Drell-Yan production is simple $p^{\uparrow} + A \to [\gamma^* \to] \ell^+ \ell^- + X$
  - quark (from polarized proton) scatters off the classical gluon field to produce a virtual photon



Kopeliovich-Raufeisen-Tarasov 01, Baier-Mueller-Schiff 04, Gelis-Jalilian-Marian 02, 03, Stasto-Xiao-Zaslavsky, 2012, Kang-Xiao 2012

- When high-energy partons scatter off the classic gluon field, the interaction is eikonal in that the projectile propagate through the target without changing their transverse position but picking up an eikonal phase

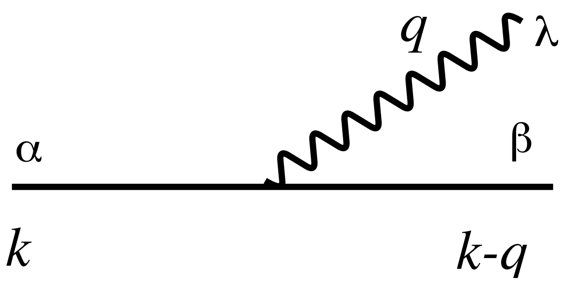
Bjorken-Kogut-Soper, 1971

$$\mathcal{S}|k^+, b, i\rangle \otimes |\mathcal{A}\rangle = U^{ij}[\mathcal{A}]|k^+, b, j\rangle \otimes |\mathcal{A}\rangle$$

$$U(x) = \mathcal{P}\exp\left\{ig_s \int_{-\infty}^{+\infty} dx^+ T^c A_c^-(x^+, x_\perp)\right\}$$

Quark splitting wave function: keep quark kt from proton

- Quark to photon splitting wave function in light-front perturbation theory $q \to q + \gamma^*$



$z = q^+/k^+$ $\epsilon_M^2 = (1-z)M^2$

  - In momentum space

$$\phi_{\alpha\beta}^{\lambda}(k,q) = \frac{1}{\sqrt{8(k-q)^+k^+q^+}} \frac{\bar{u}_\beta(k-q)\gamma_\mu\epsilon^\mu(q,\lambda)u_\alpha(k)}{(k-q)^- + q^- - k^-}$$

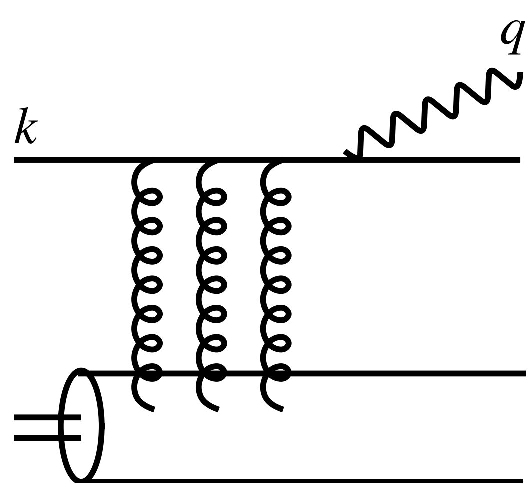
  - In transverse coordinate space: $\psi_{\alpha\beta}^{\lambda}(k,q^+,r) = \int d^2q_\perp e^{iq_\perp\cdot r}\phi_{\alpha\beta}^{\lambda}(k,q)$

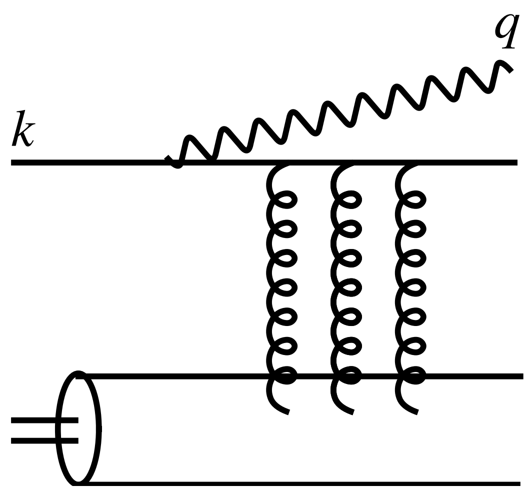
$$\psi_{\alpha\beta}^{T\,\lambda}(k,q^+,r) = 2\pi\sqrt{\frac{2}{q^+}}e^{izk_\perp\cdot r}i\epsilon_M K_1(\epsilon_M|r|)\begin{cases}\frac{r\cdot\epsilon_\perp^1}{|r|}\left[\delta_{\alpha-}\delta_{\beta-} + (1-z)\delta_{\alpha+}\delta_{\beta+}\right], & \lambda = 1,\\ \frac{r\cdot\epsilon_\perp^2}{|r|}\left[\delta_{\alpha+}\delta_{\beta+} + (1-z)\delta_{\alpha-}\delta_{\beta-}\right], & \lambda = 2.\end{cases}$$

$$\psi_{\alpha\beta}^{L}(k,q^+,r) = 2\pi\sqrt{\frac{2}{q^+}}e^{izk_\perp\cdot r}(1-z)MK_0(\epsilon_M|r|)\delta_{\alpha\beta}$$

The multiple scattering could happen before or after

- The interaction with the target could happen before or after the splitting of the virtual photon





- The differential cross section for $q + A \to \gamma^* + X$

$$\frac{d\sigma(qA \to \gamma^* X)}{dq^+ d^2 q_\perp} = \alpha_{\rm em} e_q^2 \int \frac{d^2 b}{(2\pi)^2} \frac{d^2 r}{(2\pi)^2} \frac{d^2 r'}{(2\pi)^2} e^{-iq_\perp \cdot (r - r')} \sum_{\alpha\beta\lambda} \psi^{*\lambda}_{\alpha\beta}(k, q^+, r' - b) \psi^\lambda_{\alpha\beta}(k, q^+, r - b)$$
$$\times \left[1 + S^{(2)}_{x_A}(v, v') - S^{(2)}_{x_A}(b, v') - S^{(2)}_{x_A}(v, b)\right]$$

  - multiple scattering is taken care of by

$$S^{(2)}_{x_A}(x, y) = \frac{1}{N_c} \left\langle \mathrm{Tr}\left(U(x) U^\dagger(y)\right)\right\rangle_{x_A}$$

Transform to momentum space

- Transform to momentum space

$$\frac{d\sigma(qA\to\gamma^* X)}{dy d^2q_\perp} = \frac{\alpha_{\rm em}}{2\pi^2}e_q^2\int d^2b d^2p_\perp F(x_A,p_\perp)\left[H_T(q_\perp,k_\perp,p_\perp,z)+H_L(q_\perp,k_\perp,p_\perp,z)\right]$$

  - Unintegrated gluon distribution (dipole gluon distribution)

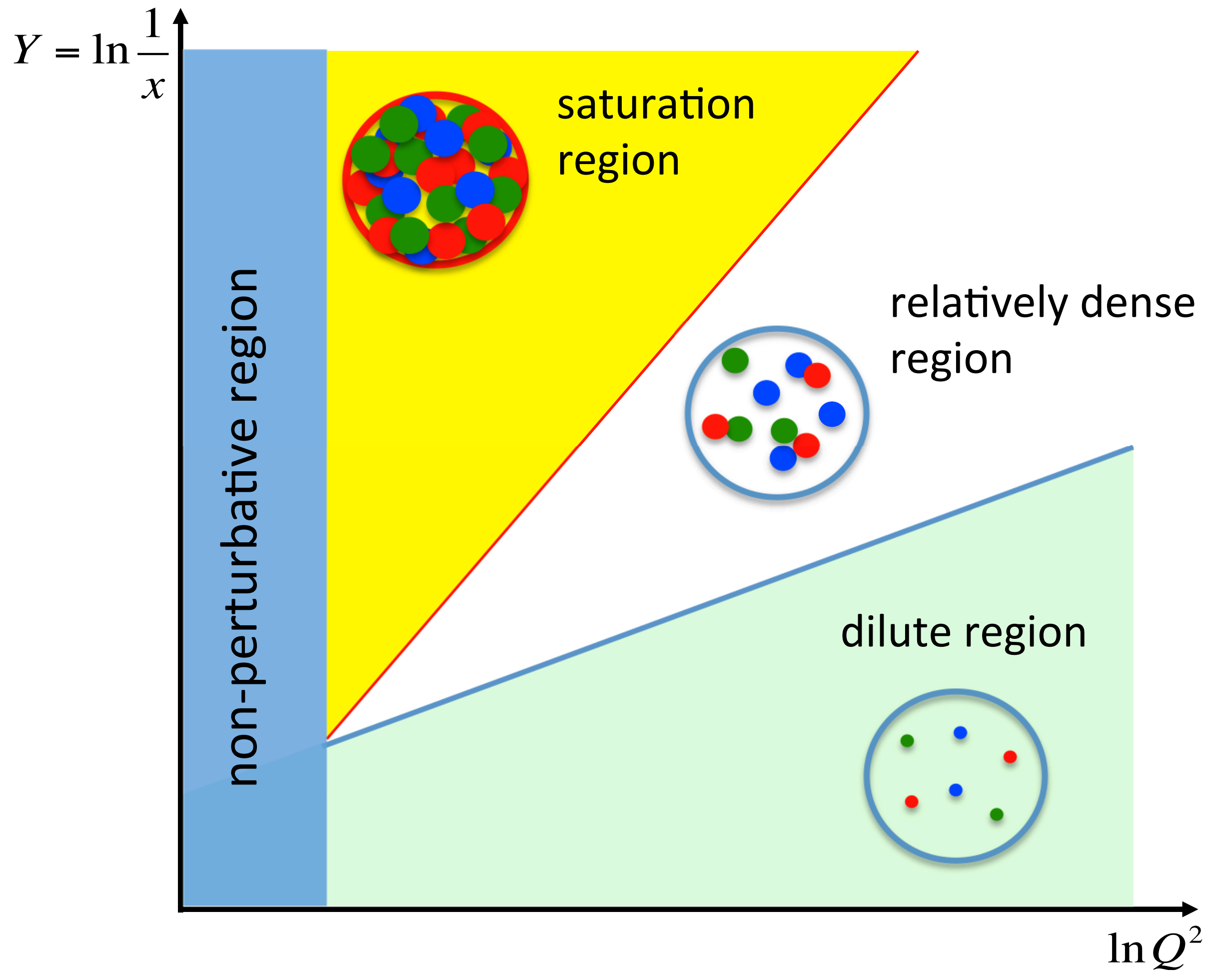
$$F(x_A,p_\perp) = \int\frac{d^2r_\perp}{(2\pi)^2}e^{ip_\perp\cdot r_\perp}\frac{1}{N_c}\left\langle {\rm Tr}\left(U(0)U^\dagger(r_\perp)\right)\right\rangle_{x_A}$$

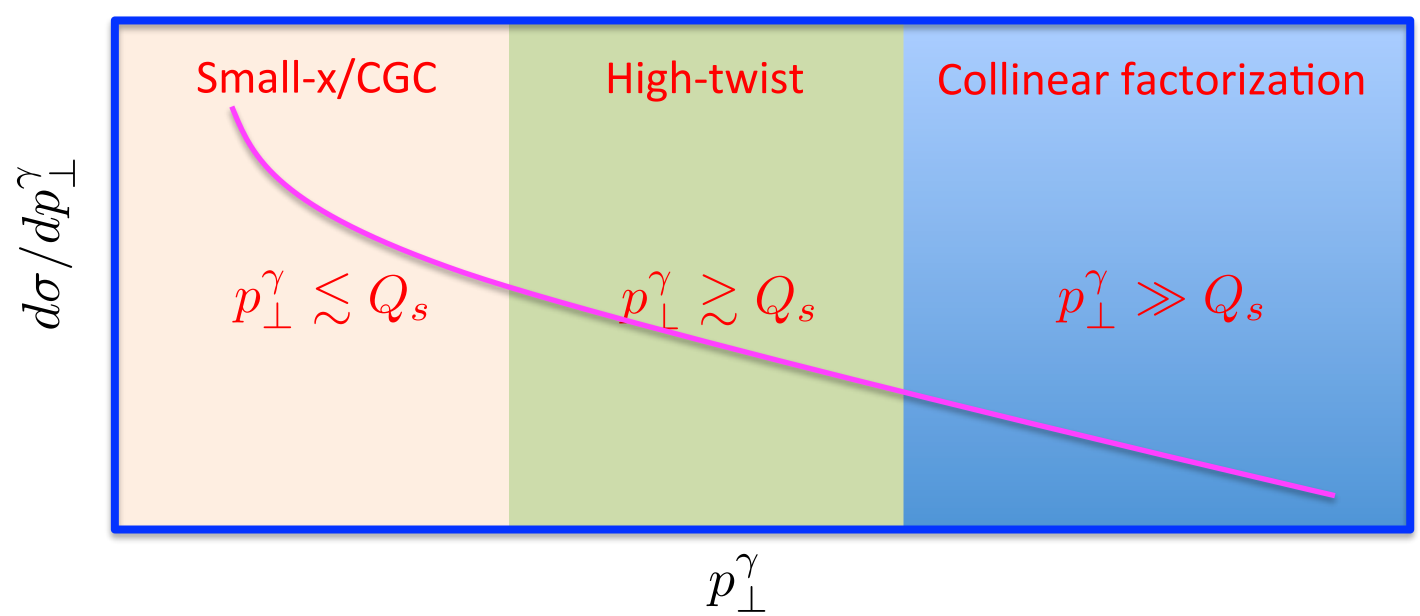
  - Hard-part functions (transverse and longitudinal polarized photon)

$$H_T(q_\perp,k_\perp,p_\perp,z) = \left[1+(1-z)^2\right]\left[\frac{q_\perp-zk_\perp}{(q_\perp-zk_\perp)^2+\epsilon_M^2}-\frac{q_\perp-zk_\perp-zp_\perp}{(q_\perp-zk_\perp-zp_\perp)^2+\epsilon_M^2}\right]^2$$

$$H_L(q_\perp,k_\perp,p_\perp,z) = 2(1-z)^2M^2\left[\frac{1}{(q_\perp-zk_\perp)^2+\epsilon_M^2}-\frac{1}{(q_\perp-zk_\perp-zp_\perp)^2+\epsilon_M^2}\right]^2$$

Unified pQCD formalism





Summary

- pQCD factorization: collinear factorization and TMD factorization
- Parton multiple scattering is very important in understanding nontrivial nuclear dependence
- Factorization at twist-4 has been verified up to one-loop order
- When parton density becomes very large, the small-x formalism should naturally come in: connection between high-twist formalism and small-x formalism should naturally match onto each other