Multiple Scattering in cold nuclear matter

Hongxi Xing (LANL)

Z. B. Kang, E. Wang, X. N. Wang and H. Xing, Phys. Rev. Lett. 112, 102001 (2014)
Z. B. Kang, Ivan Vitev and H. Xing, Phys. Rev. D88, 054010 (2013)
Z. B. Kang, Ivan Vitev and H. Xing, arXiv: 1406.xxxx (2014)

JET Collaboration Meeting, UC Davis, June 17 - 18, 2014

Outline

* Introduction
* Factorization of multiple scatterings in CNM

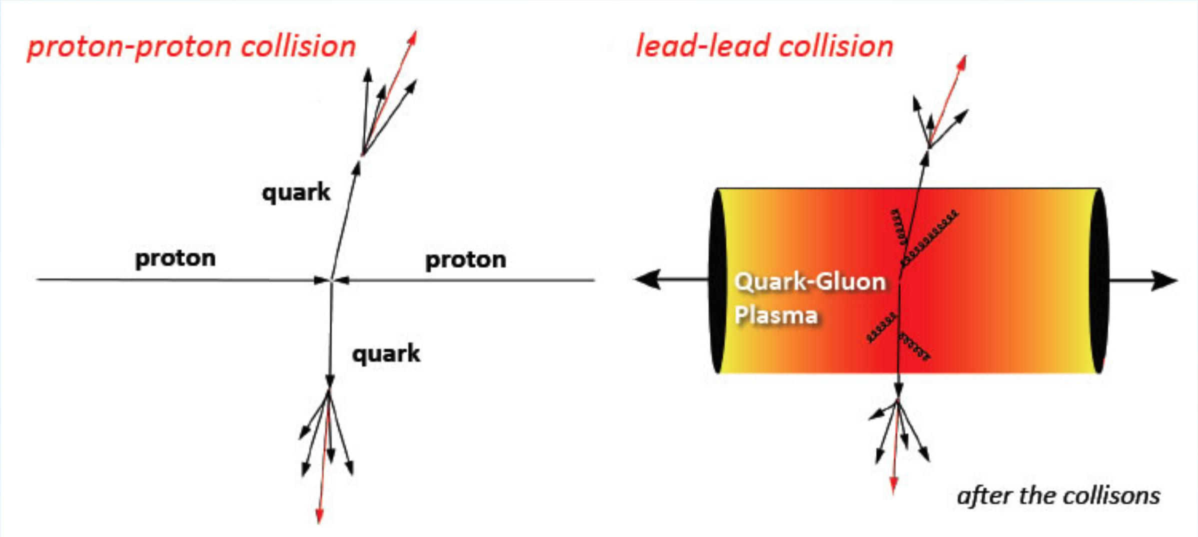
  1. SIDIS 2. DY
* Multiple scatterings in p+A collisions

  1. Light hadron production

  2. Heavy meson production
* Summary

Introduction

- Factorization in heavy ion ?

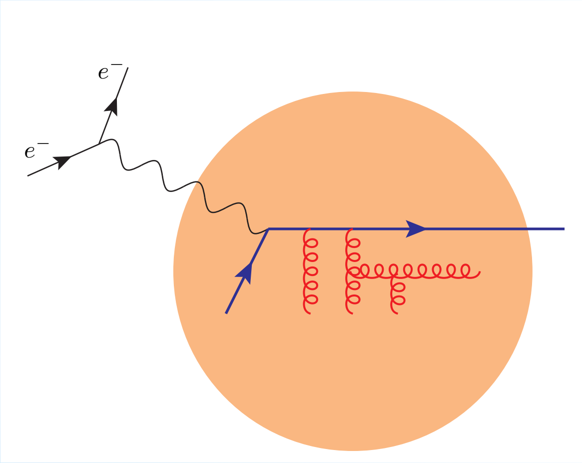


PP: $$d\sigma = \sum_{abcd} f_{a/A}(x_a) \otimes f_{b/B}(x_b) \otimes d\hat{\sigma}_{ab\to cd} \otimes D_{h/c}(z)$$

AA: $$d\sigma = \sum_{abcd} f_{a/A}(x_a) \otimes f_{b/B}(x_b) \otimes d\hat{\sigma}_{ab\to cd} \otimes \tilde{D}_{h/c}(z_m)$$

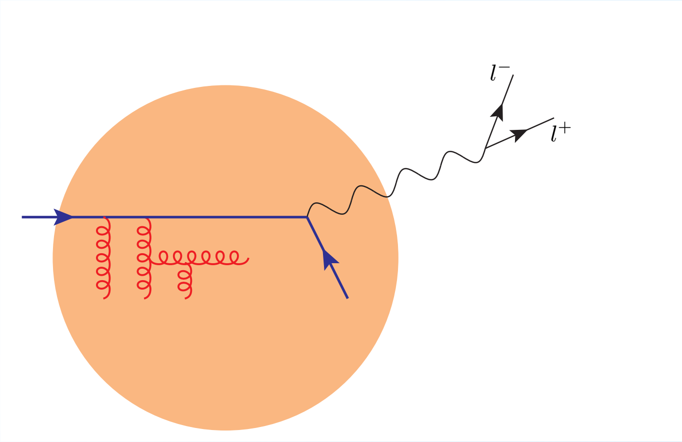
- Where should we start?
  - Answer: Multiple Scattering in Cold Nuclear Matter (CNM)
  - Two cleanest processes to study multiple scattering

SIDIS



Final state multiple scattering

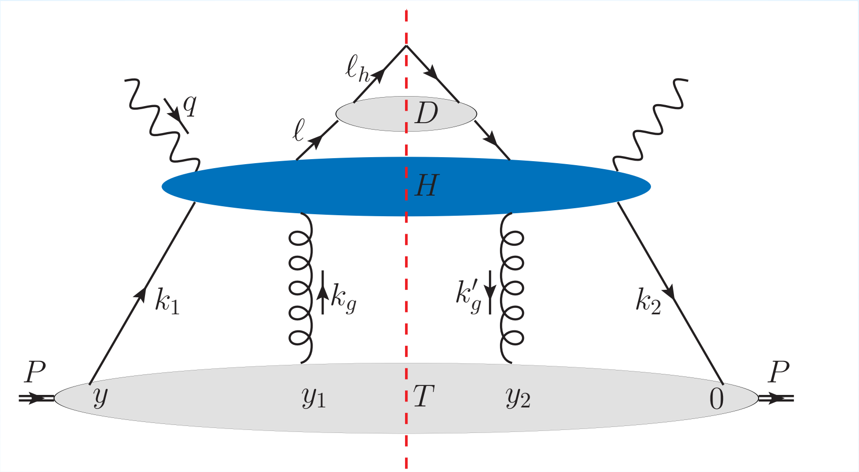
DY



Initial state multiple scattering

  - Single inclusive particle production in p+A collisions, involve both initial state and final state multiple scattering.

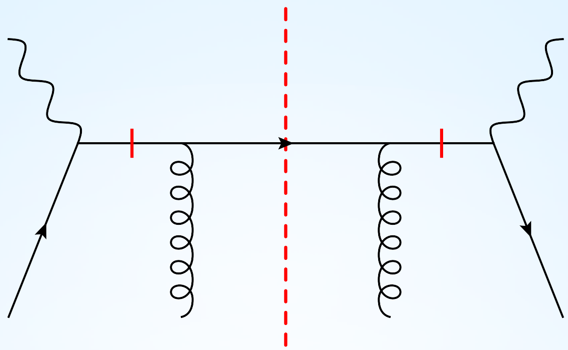
Double scattering in SIDIS (Twist-4)



Transverse momentum broadening (TMB)

$$\Delta\langle \ell_T^2\rangle = \langle \ell_T^2\rangle^{eA} - \langle \ell_T^2\rangle^{eN} \approx \frac{\int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eA}^D}{dQ^2 d\ell_T^2}}{\frac{d\sigma_{eA}}{dQ^2}}$$

Twist-4 leading order



Guo, 1998
Guo, Qiu 2000

- Leading contribution to broadening of hadron

$$\Delta\langle \ell_{hT}^2 \rangle = \left(\frac{4\pi^2 \alpha_s}{N_c} z_h^2 \right) \frac{\sum_q e_q^2 T_F(x_B, 0, 0) D_{h/q}(z_h)}{\sum_q e_q^2 f_{q/A}(x_B) D_{h/q}(z_h)}$$

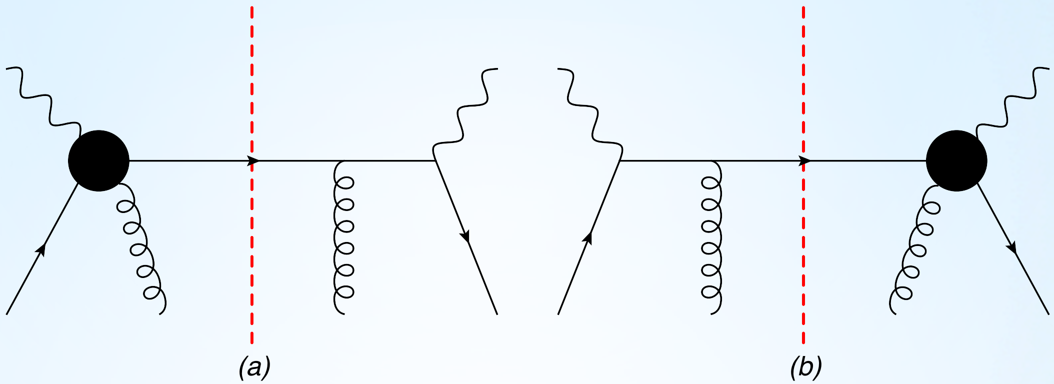
T-4 q-g correlation:

$$T_F(x_1, x_2, x_3) = \int \frac{dy^-}{2\pi} e^{ix_1 p^+ y^-} \int \frac{dy_1^- dy_2^-}{4\pi} e^{ix_2 p^+ (y_1^- - y_2^-)} e^{ix_3 p^+ y_2^-} \times \langle A | \bar{\psi}_q(0) \gamma^+ F_\sigma^+(y_2^-) F^{\sigma +}(y_1^-) \psi_q(y^-) | A \rangle \theta(y_2^-) \theta(y_1^- - y^-)$$

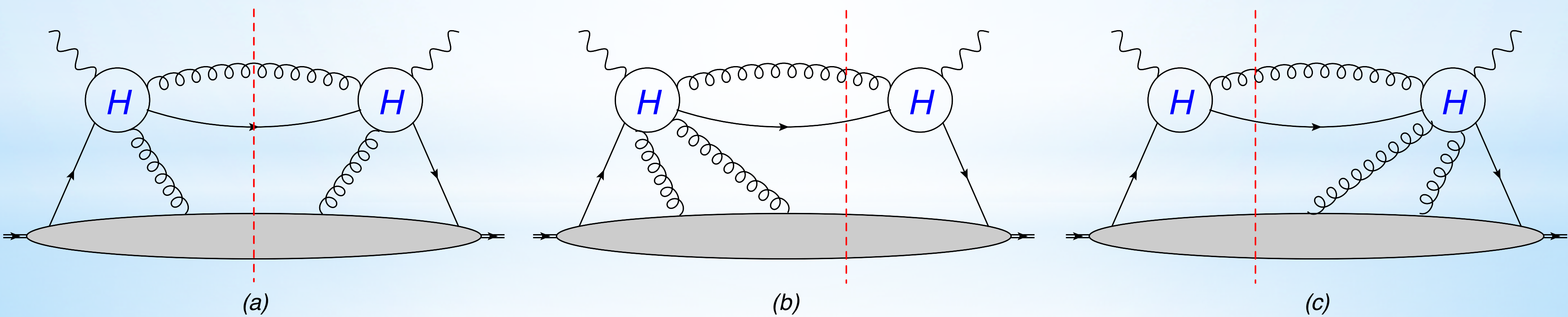
Provide a way to measure the T-4 quark-gluon correlation function.

Twist-4 NLO – Feynman diagram

- Virtual



- Real



central-cut   right-cut   left-cut

- Soft divergence (double pole $\propto \frac{1}{\epsilon^2}$)

Real + virtual → 0

- collinear divergences (single pole $\propto \frac{1}{\epsilon}$)

Collinear to FS

$$-\frac{1}{\epsilon}\delta(1-\hat{x})T_F(x,0,0)P_{qq}(\hat{z})$$

$\overline{MS}$

$$D_q(z_h,\mu^2) = D_q^0(z_h) + \frac{\alpha_s}{2\pi}\int \frac{dz}{z}\left(-\frac{1}{\hat{\epsilon}}\right)P_{qq}(\hat{z})D_q(z)$$

DGLAP

Collinear to IS

$$-\frac{1}{\epsilon}\delta(1-\hat{z})\left[T_F(x,0,0)P_{qq}(\hat{x}) + P_{qg\to qg}(\hat{x})\otimes T_F(x,x,x_B)\right]$$

$\overline{MS}$

$$T_F(x_B,0,0,\mu^2) = T_F^{(0)}(x_B,0,0) - \frac{\alpha_s}{2\pi}\frac{1}{\hat{\epsilon}}\int_{x_B}^1 \frac{dx}{x}\left[P_{qq}(\hat{x})T_F(x,0,0) + P_{qg\to qg}(\hat{x})\otimes T_F(x,x,x_B)\right]$$

Factorization at twist-4 in SIDIS

- Evolution equation for TF

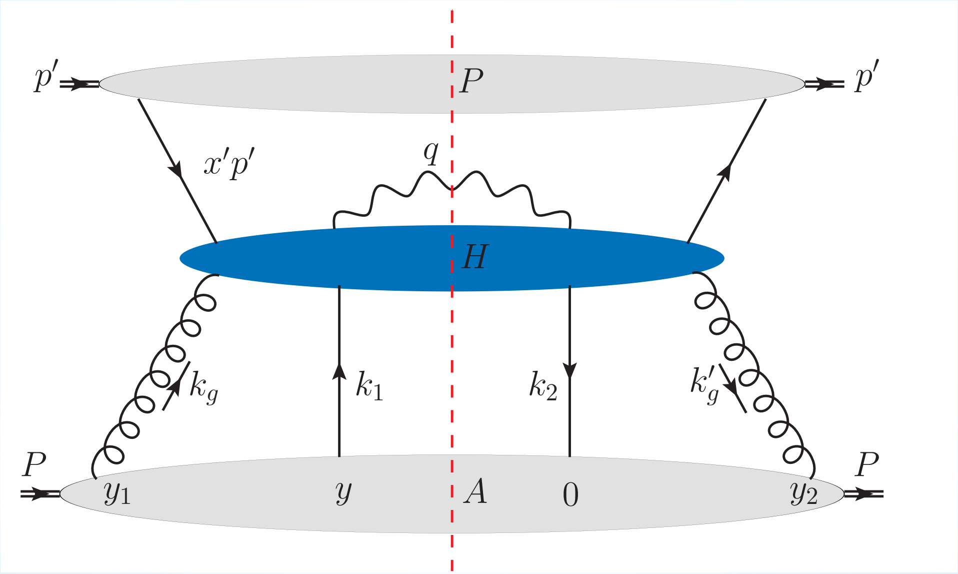
$$\mu^2 \frac{\partial}{\partial \mu^2} T_F(x_B, 0, 0, \mu^2) = \frac{\alpha_s}{2\pi} \int_{x_B}^{1} \frac{dx}{x} \left[P_{qq}(\hat{x}) T_F(x, 0, 0, \mu^2) + P_{qg \to qg}(\hat{x}) \otimes T_F(x, x, x_B, \mu^2) \right]$$

- Factorization

$$\frac{d\langle \ell_{hT}^2 \sigma \rangle}{dz_h} \propto D_{q/h}(z, \mu^2) \otimes H^{LO}(x, z) \otimes T_F(x, 0, 0, \mu^2) + \frac{\alpha_s}{2\pi} D_{q/h}(z, \mu^2) \otimes H^{NLO}(x, z, \mu^2) \otimes T_F(x, 0, 0, \mu^2)$$

Multiple scattering hard part coefficients and medium properties can be factorized!!!

Verification in Drell-Yan



Factorization in Drell-Yan at twist-4

- Soft divergence cancel (real + virtual)
- collinear divergence
  - Redefinition of beam PDF

$$f_q(x',\mu^2) = f_q^0(x') + \frac{\alpha_s}{2\pi}\int \frac{d\xi}{\xi}\left(-\frac{1}{\hat{\epsilon}}\right) P_{qq}(z) f_q(\xi)$$

  - Redefinition of nuclear T-4 gluon-quark correlation function

$$T_F(x_B,0,0,\mu^2) = T_F^{(0)}(x_B,0,0) - \frac{\alpha_s}{2\pi}\frac{1}{\hat{\epsilon}}\int_{x_B}^{1}\frac{dx}{x}\left[P_{qq}(\hat{x})T_F(x,0,0) + P_{qg\to qg}(\hat{x})\otimes T_F(x,x,x_B)\right]$$

Exactly the same as that in SIDIS, it is universal!

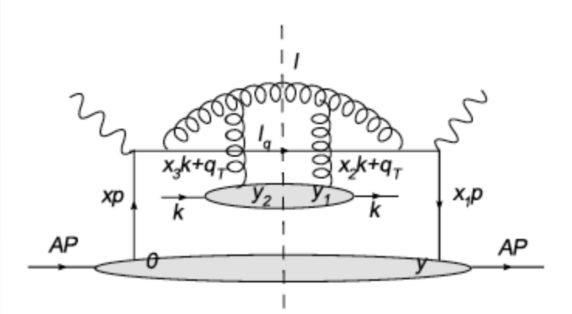
- Transverse momentum weighted cross section

$$\frac{d\langle q_T^2\sigma\rangle^{DY}}{dQ^2} = \sigma_0^{DY}\int\frac{dx'}{x'}f_{\bar{q}}(x',\mu^2)\int\frac{dx}{x}T_F(x,0,0,\mu^2)\delta(1-z)$$
$$+\sigma_0^{DY}\frac{\alpha_s}{2\pi}\int\frac{dx'}{x'}f_{\bar{q}}(x',\mu^2)\int\frac{dx}{x}H^{NLO}(z,x)\otimes T_F(x,x,x_B,\mu^2)$$

Discussion - Evolution of jet transport parameter

- Related to jet transport parameter

J. Casalderrey-Solana and X.-N. Wang (2008)



$$T_F(x_B,0,0,\mu^2) \approx \frac{N_c}{4\pi^2\alpha_s} f_{q/A}(x_B,\mu^2) \int dy^- \hat{q}(\mu^2,y^-)$$

$$\hat{q}(\mu^2,y^-) = \frac{4\pi^2\alpha_s C_R}{N_c^2-1}\rho_N^A(y^-)xf_{g/N}(x)$$

- Evolution equation of jet transport parameter

$$\mu^2\frac{\partial}{\partial\mu^2}T_F(x_B,0,0,\mu^2) = \frac{\alpha_s}{2\pi}\int_{x_B}^1 \frac{dx}{x}\left[P_{qq}(\hat{x})T_F(x,0,0,\mu^2) + P_{qg\to qg}(\hat{x})\otimes T_F(x,x,x_B,\mu^2)\right]$$

$$P_{qg\to qg}(\hat{x})\otimes T_F(x,x,x_B)$$
$$= C_A\left[\frac{2}{(1-\hat{x})_+}T(x_B,x-x_B,0) - \frac{1}{2}\frac{1+\hat{x}}{(1-\hat{x})_+}\left(T(x,0,x_B-x)+T(x_B,x-x_B,x-x_B)\right)\right]$$

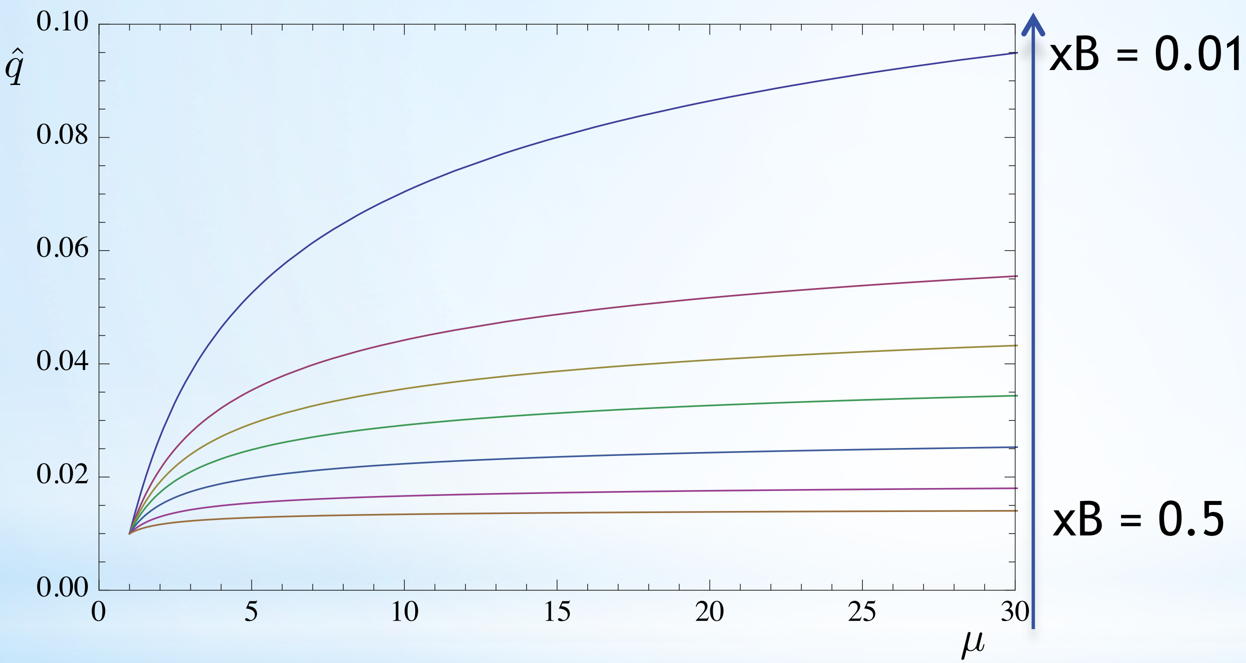
1. Large-xB limit (xB→1, LPM interference regime):

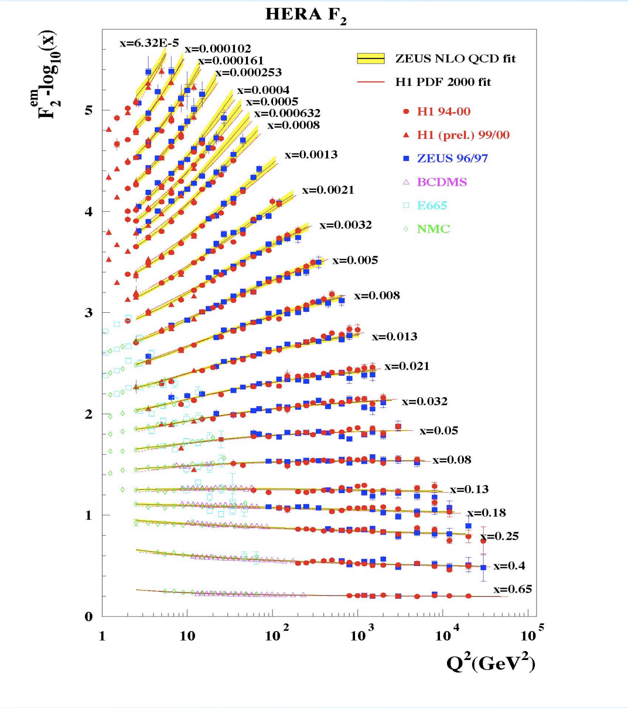
$$\mu^2\frac{\partial\hat{q}(\mu^2)}{\partial\mu^2} = 0$$

Scaling in large Bjorken-x

2. Evolution of qhat in intermediate-x

$$\frac{\partial \hat{q}(\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi} C_A \ln(1/x_B)\hat{q}(\mu^2) \quad \Rightarrow \quad \hat{q}(\mu^2) = \hat{q}(\mu_0^2) Exp\left[\frac{\alpha_s}{2\pi} C_A \ln(1/x_B)\ln(\mu^2/\mu_0^2)\right]$$



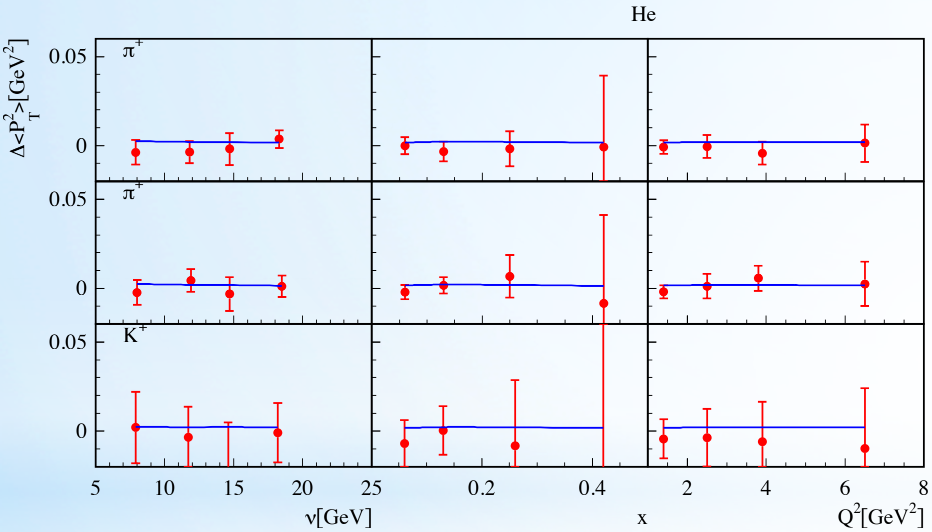


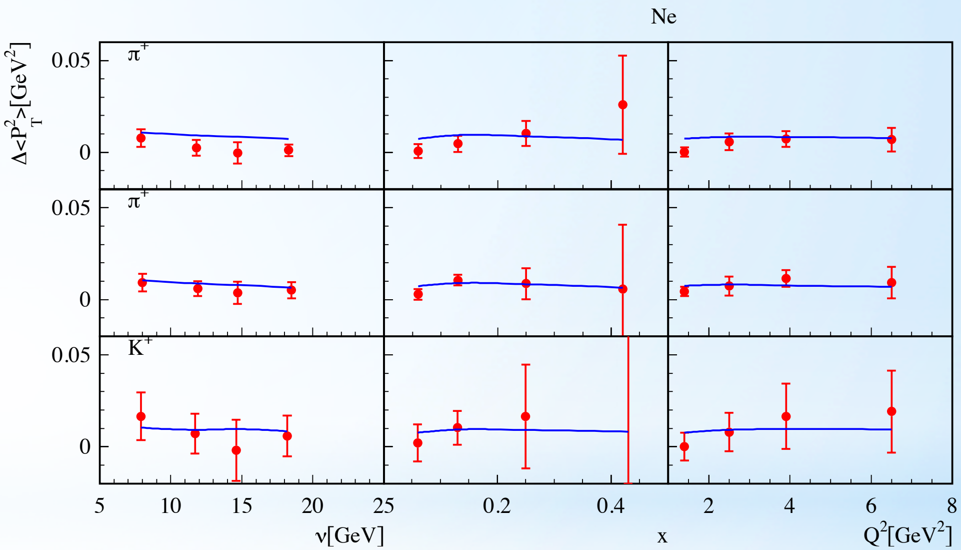
1. mu-dependence → Scaling violation!
2. Energy dependence → consistent with earlier expectation

J. Casalderrey-Solana and X.-N. Wang (2008)

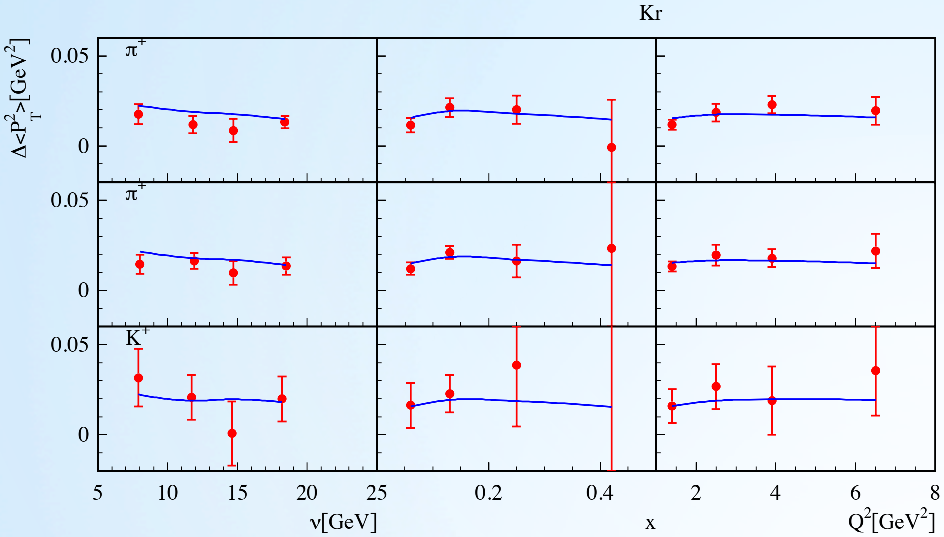
- Compare to HERMES data (LO with scale dependent q-g correlation function)

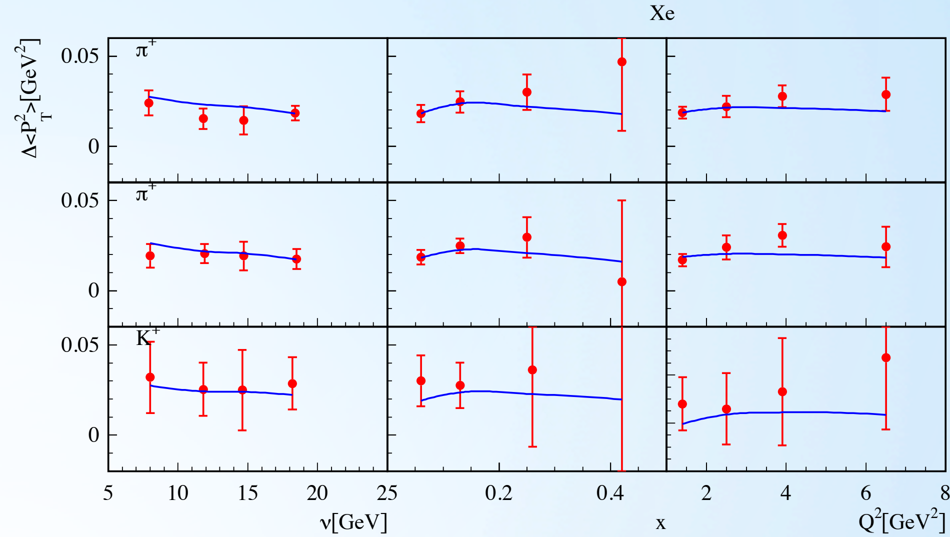
$$\hat{q}(\mu_0 = 1) = 0.015 GeV^2/fm$$

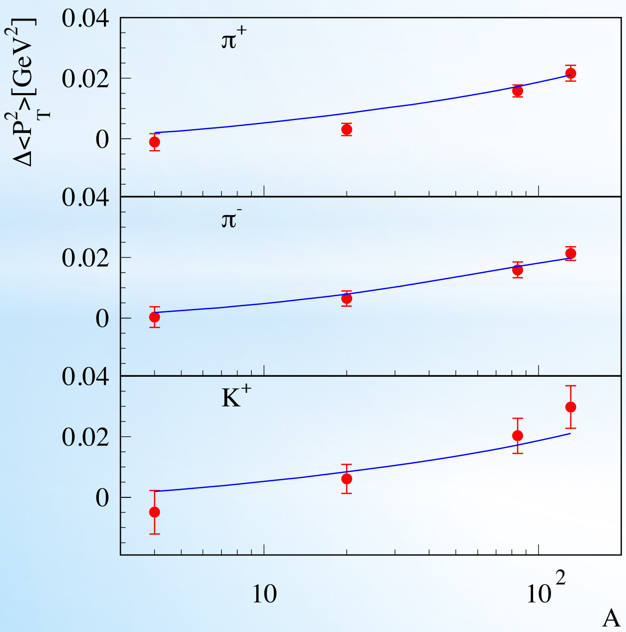




- Compare to HERMES data (LO with scale dependent q-g correlation function)





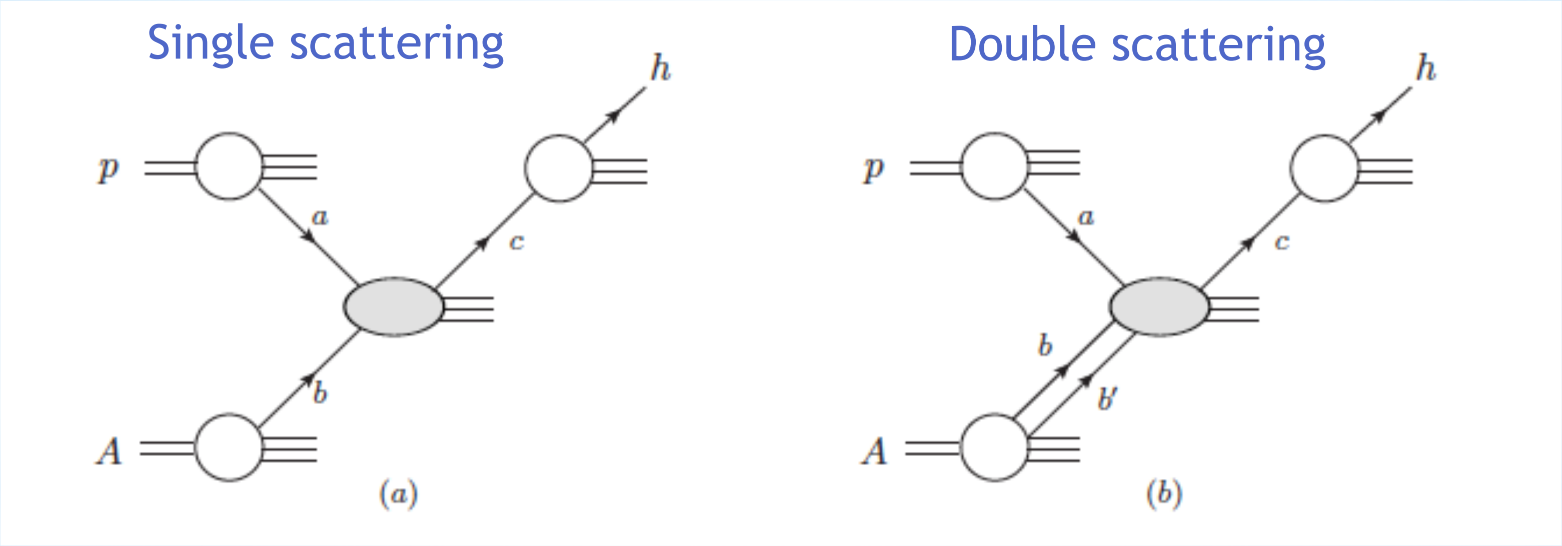


All kinematic dependence can be described quite well !

Single inclusive hadron production in p+A collisions

- Cross section expansion

$$d\sigma_{pA\to hX} = d\sigma^{(S)}_{pA\to hX} + d\sigma^{(D)}_{pA\to hX} + \cdots$$

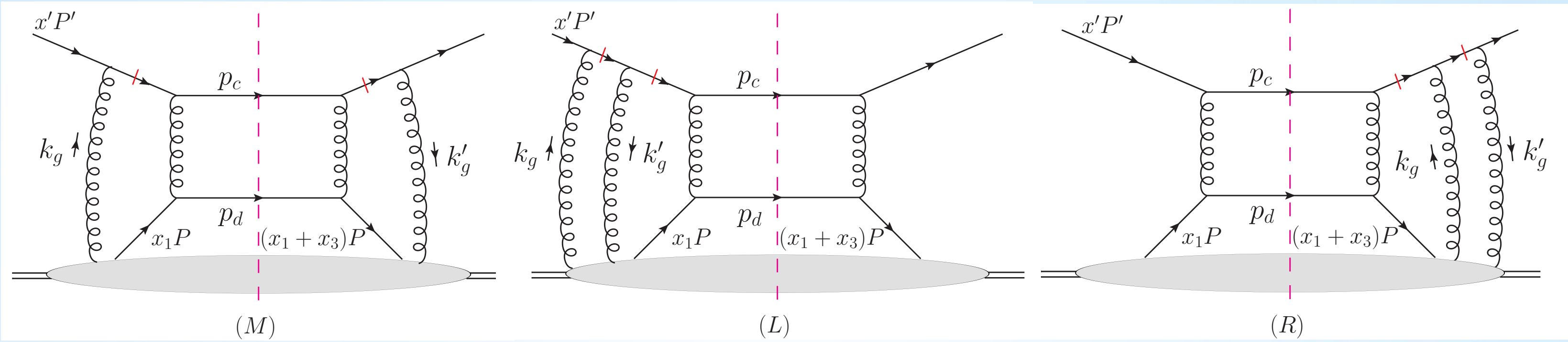


- Single scattering contribution

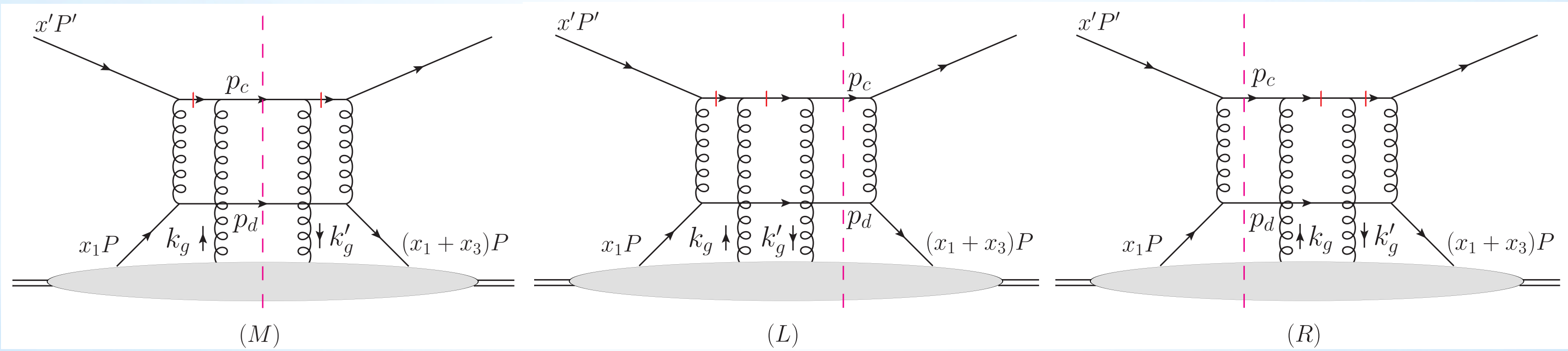
$$E_h \frac{d\sigma^{(S)}}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dz}{z^2} D_{c\to h}(z) \int \frac{dx'}{x'} f_{a/p}(x') \int \frac{dx}{x} f_{b/A}(x) H^U_{ab\to cd}(\hat{s},\hat{t},\hat{u}) \delta(\hat{s}+\hat{t}+\hat{u})$$

- **Double scattering Feynman diagrams** ($qq' \to qq'$ as an example)

Initial state double scattering



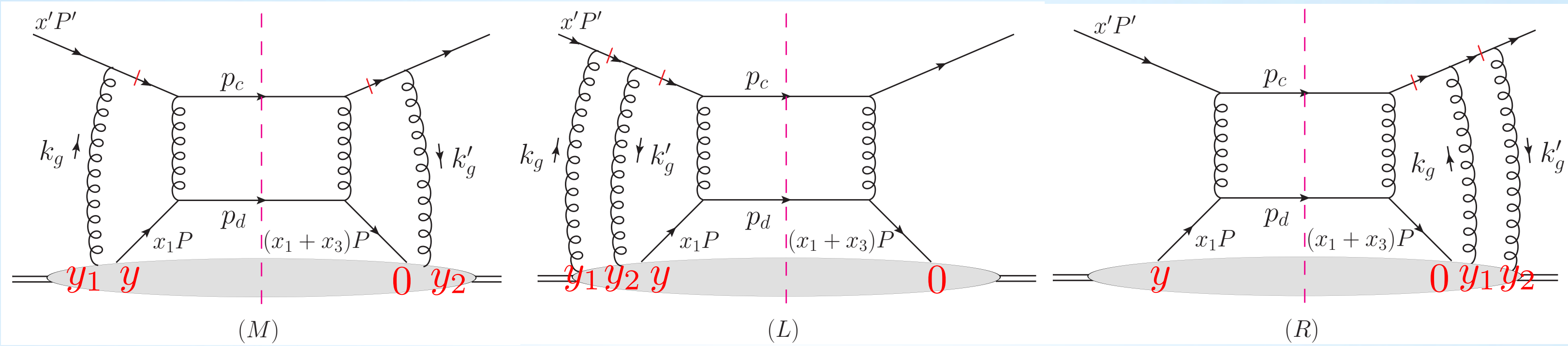
Final state double scattering



- **Double scattering cross section**

$$E_h \frac{d\sigma^{(D)}}{d^3P_h} \propto \int \frac{dz}{z^2} D_{c\to h}(z) \int \frac{dx'}{x'} f_{a/p}(x') \int dx_1 dx_2 dx_3 T(x_1, x_2, x_3) \left(-\frac{1}{2} g^{\rho\sigma}\right) \left[\frac{1}{2} \frac{\partial^2}{\partial k_\perp^\rho \partial k_\perp^\sigma} H(x_1, x_2, x_3, k_\perp)\right]_{k_\perp}$$

- Contact contribution



$\theta(y^- - y_1^-)\theta(-y_2^-)$ $\quad -\theta(y_2^- - y_1^-)\theta(y^- - y_2^-)$ $\quad -\theta(y_1^- - y_2^-)\theta(-y_1^-)$

$$\propto \int \frac{dy^-}{2\pi} e^{ixP^+y^-} \int \frac{dy_1^- dy_2^-}{2\pi} (y_1^- - y_2^-)^2 \langle P| F_\alpha{}^+(y_2^-)\bar{\psi}_q(0)\gamma^+\psi_q(y^-)F^{+\alpha}(y_1^-)|P\rangle$$
$$\times H(x,0,0,0)\left[\theta(y_2^- - y_1^-)\theta(y^- - y_2^-) + \theta(y_1^- - y_2^-)\theta(-y_1^-) - \theta(y^- - y_1^-)\theta(-y_2^-)\right]$$

position constrain: $|y^-| > |y_1^-| > |y_2^-|$

$e^{ixP^+y^-} \rightarrow \quad y^- \sim \frac{1}{xP^+} \xrightarrow{x \to \mathcal{O}(1)} \quad y^- \to 0 \quad y_1^-, y_2^- \to 0$

All of the y-integrations are localized, therefore can be neglected due to the lack of nuclear size enhancement.

- Final contribution (incoherent multiple scattering)

$$E_h \frac{d\sigma^{(D)}}{d^3P_h} = \left(\frac{8\pi^2\alpha_s}{N_c^2-1}\right)\frac{\alpha_s^2}{S}\sum_{a,b,c}\int\frac{dz}{z^2}D_{c\to h}(z)\int\frac{dx'}{x'}f_{a/p}(x')\int\frac{dx}{x}\delta(\hat{s}+\hat{t}+\hat{u})$$

$$\times\sum_{i=I,F}\left[x^2\frac{\partial^2 T_{b/A}^{(i)}(x)}{\partial x^2} - x\frac{\partial T_{b/A}^{(i)}(x)}{\partial x} + T_{b/A}^{(i)}(x)\right] c^i H^i_{ab\to cd}(\hat{s},\hat{t},\hat{u})$$

Only central-cut contributes.

Double scattering hard factor:

$$c^I = -\frac{1}{\hat{t}} - \frac{1}{\hat{s}}$$

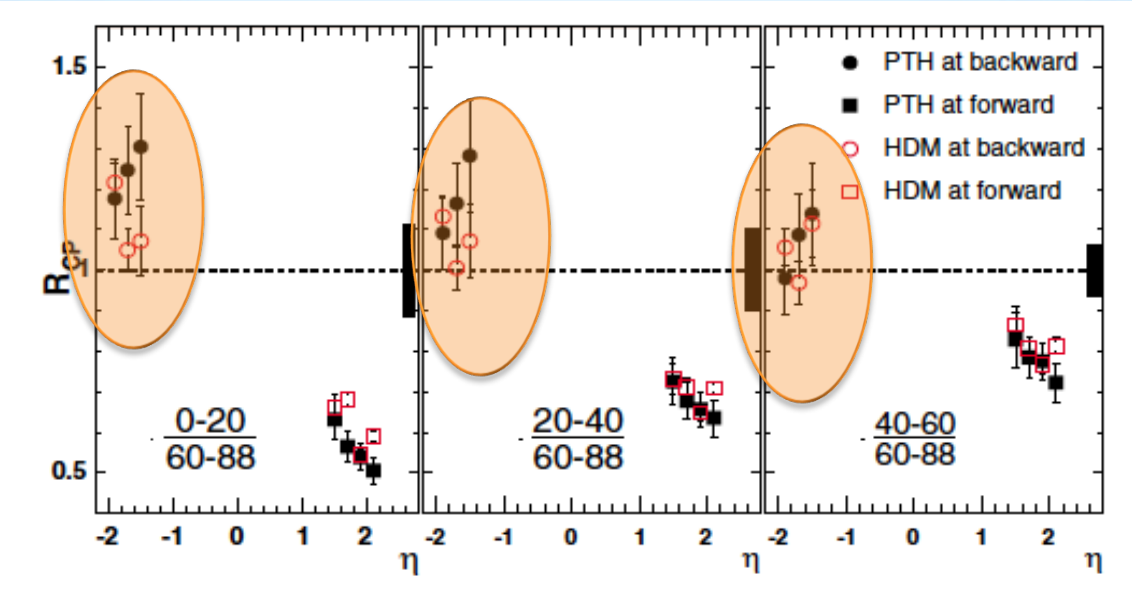
$$c^F = -\frac{1}{\hat{t}} - \frac{1}{\hat{u}}$$

Double scattering color strength:

$$H^I_{ab\to cd} = \begin{cases} C_F H^U_{ab\to cd} & \text{a=quark} \\ C_A H^U_{ab\to cd} & \text{a=gluon} \end{cases}$$ (a: incoming)

$$H^F_{ab\to cd} = \begin{cases} C_F H^U_{ab\to cd} & \text{c=quark} \\ C_A H^U_{ab\to cd} & \text{c=gluon} \end{cases}$$ (c: outgoing)

- Enhancement in large-x regime



PHENIX PRL 94, 082302 (2005)

$$E_h \frac{d\sigma^{(D)}}{d^3P_h} = \left(\frac{8\pi^2\alpha_s}{N_c^2-1}\right)\frac{\alpha_s^2}{S}\sum_{a,b,c}\int\frac{dz}{z^2}D_{c\to h}(z)\int\frac{dx'}{x'}f_{a/p}(x')\int\frac{dx}{x}\delta(\hat{s}+\hat{t}+\hat{u})$$

$$\times\sum_{i=I,F}\left[x^2\frac{\partial^2 T^{(i)}_{b/A}(x)}{\partial x^2} - x\frac{\partial T^{(i)}_{b/A}(x)}{\partial x} + T^{(i)}_{b/A}(x)\right] c^i H^i_{ab\to cd}(\hat{s},\hat{t},\hat{u})$$

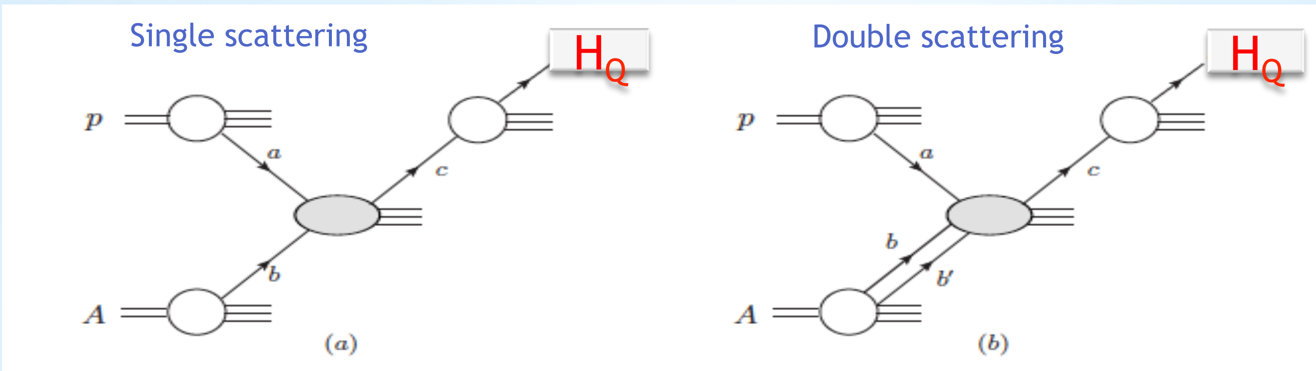
POSITIVE POSITIVE

In the large-x region, incoherent multiple scattering leads to the enhancement.

Heavy meson production in p+A collisions

- Cross section expansion

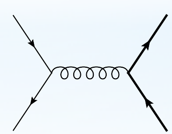
$$d\sigma_{pA\to HX} = d\sigma^{(S)}_{pA\to HX} + d\sigma^{(D)}_{pA\to HX} + \cdots$$



- Single scattering contribution

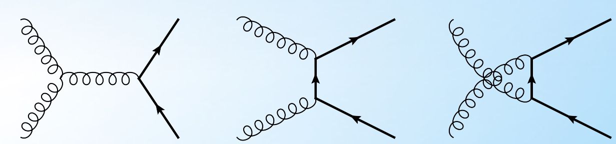
$$E_h \frac{d\sigma^{(S)}}{d^3P_h} = \frac{\alpha_s^2}{S}\sum_{a,b}\int \frac{dz}{z^2} D_{c\to H}(z)\frac{dx'}{x'} f_{a/p}(x')\int \frac{dx}{x} f_{b/A}(x) H^U_{ab\to c}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u})$$

$$H^U_{q\bar{q}\to Q\bar{Q}} = \frac{N_c^2-1}{2N_c^2}\frac{\hat{t}^2+\hat{u}^2+2m_c^2\hat{s}}{\hat{s}^2}$$



(a)

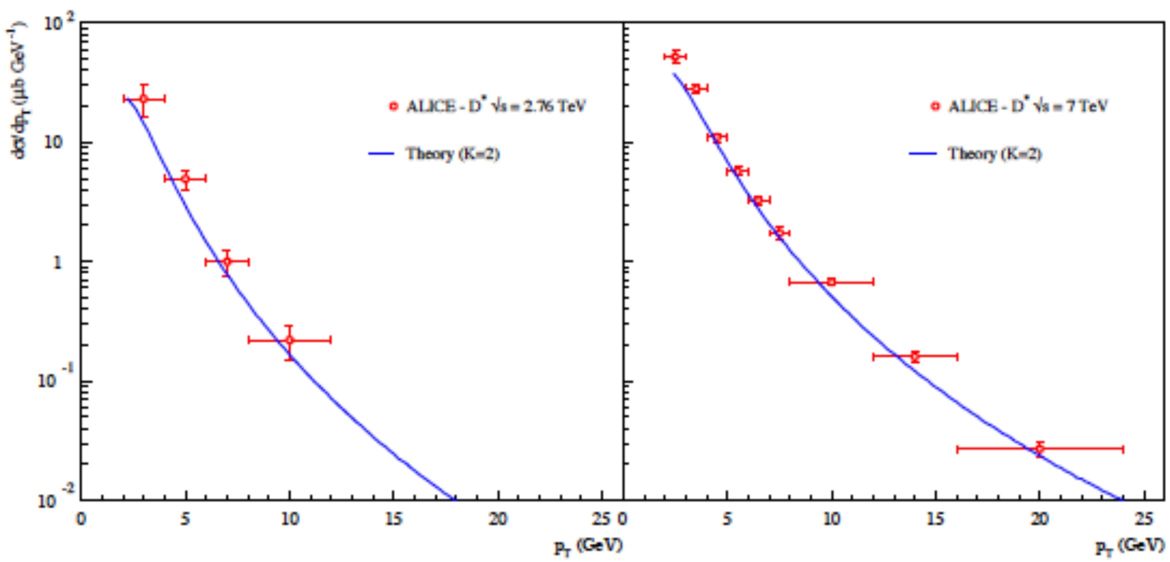
$$H^U_{gg\to\to Q\bar{Q}} = \frac{1}{2N_c}\left(\frac{1}{\hat{t}\hat{u}} - \frac{2N_c^2}{N_c^2-1}\frac{1}{\hat{s}^2}\right)\left(\hat{t}^2+\hat{u}^2+4m_c^2\hat{s} - \frac{4m_c^2\hat{s}^2}{\hat{t}\hat{u}}\right)$$

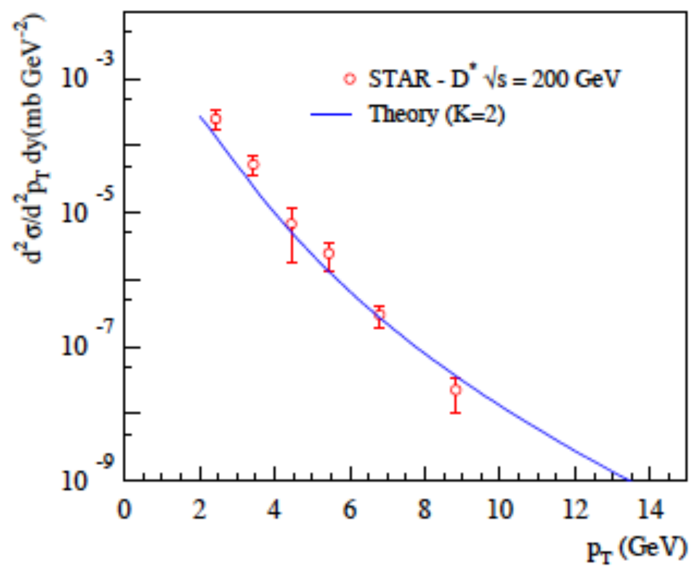


(b)

- Compare to experimental data for D-meson production

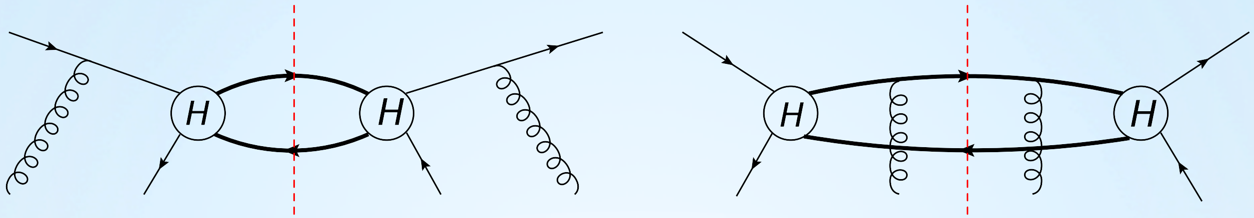
$$E_h \frac{d\sigma^{(S)}}{d^3P_h} = K_{NLO} \frac{\alpha_s^2}{S} \sum_{a,b} \int \frac{dz}{z^2} D_{c\to H}(z) \frac{dx'}{x'} f_{a/p}(x') \int \frac{dx}{x} f_{b/A}(x) H^U_{ab\to c}(\hat{s},\hat{t},\hat{u}) \delta(\hat{s}+\hat{t}+\hat{u})$$





Good descriptions to LHC and RHIC data with $K_{NLO} = 2$ for D*, D+ and D0.

- Double scattering – annihilation channel



$$E_h \frac{d\sigma^{(D)}}{d^3P_h}\Big|_{q\bar{q}\to Q\bar{Q}} = \frac{8\pi^2\alpha_s}{N_c^2-1}\frac{\alpha_s^2}{S}\sum_q \int \frac{dz}{z^2} D_{Q\to H}(z) \frac{dx'}{x'} f_{q/p}(x') \int \frac{dx}{x} H^U_{q\bar{q}\to Q\bar{Q}}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u})$$

$$\times \sum_{i=I,F}\left(x^2\frac{\partial^2 T_{\bar{q}/A}(x)}{\partial x^2} D_2^{qi} - x\frac{\partial T_{\bar{q}/A}(x)}{\partial x} D_1^{qi} + T_{\bar{q}/A}(x) D_0^{qi}\right)$$

Double scattering factor:

Initial

$$D_2^{qI} = C_F\left[-\frac{1}{\hat{t}} - \frac{1}{\hat{s}} - \frac{m_c^2}{\hat{t}^2}\right]$$

$$D_1^{qI} = C_F\left[-\frac{1}{\hat{t}} - \frac{1}{\hat{s}} - 2\frac{m_c^2}{\hat{t}^2}\frac{(\hat{t}-\hat{u})^2+4m_c^2\hat{s}}{2m_c^2\hat{s}+\hat{t}^2+\hat{u}^2}\right]$$

$$D_0^{qI} = C_F\left[-\frac{1}{\hat{t}} - \frac{1}{\hat{s}} - 2\frac{m_c^2}{\hat{t}^2}\frac{(\hat{t}-\hat{u})^2-\hat{t}\hat{u}+6m_c^2\hat{s}}{2m_c^2\hat{s}+\hat{t}^2+\hat{u}^2}\right]$$

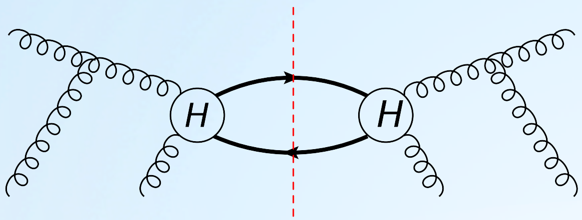
Final

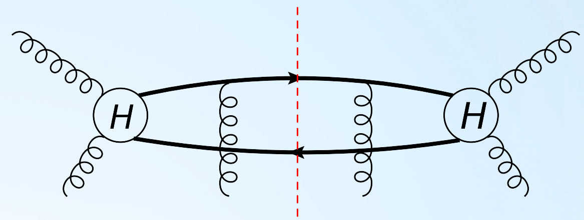
$$D_2^{qF} = C_F\left[-\frac{1}{\hat{t}} - \frac{1}{\hat{u}} - \frac{m_c^2\hat{s}^2}{\hat{t}^2\hat{u}^2}\right]$$

$$D_1^{qF} = C_F\left[-\frac{1}{\hat{t}} - \frac{1}{\hat{u}} - 2\frac{m_c^2\hat{s}^2}{\hat{t}^2\hat{u}^2}\frac{(\hat{t}-\hat{u})^2+4m_c^2\hat{s}}{2m_c^2\hat{s}+\hat{t}^2+\hat{u}^2}\right]$$

$$D_0^{qF} = C_F\left[-\frac{1}{\hat{t}} - \frac{1}{\hat{u}} - 2\frac{m_c^2\hat{s}^2}{\hat{t}^2\hat{u}^2}\frac{(\hat{t}-\hat{u})^2-\hat{t}\hat{u}+6m_c^2\hat{s}}{2m_c^2\hat{s}+\hat{t}^2+\hat{u}^2}\right]$$

- Double scattering – fusion channel





$$E_h \frac{d\sigma^{(D)}}{d^3P_h}\bigg|_{gg\to Q\bar{Q}} = \frac{8\pi^2\alpha_s}{N_c^2-1}\frac{\alpha_s^2}{S}\int\frac{dz}{z^2}D_{Q\to H}(z)\frac{dx'}{x'}f_{g/p}(x')\int\frac{dx}{x}H^U_{gg\to Q\bar{Q}}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u})$$

$$\times\sum_{i=I,F}\left(x^2\frac{\partial^2 T_{g/A}(x)}{\partial x^2}D_2^{gi} - x\frac{\partial T_{g/A}(x)}{\partial x}D_1^{gi} + T_{g/A}(x)D_0^{gi}\right)$$

Double scattering factor:

Initial

$$D_2^{gI} = C_A\left[-\frac{1}{\hat{t}} - \frac{1}{\hat{s}} - \frac{m_c^2}{\hat{t}^2}\right]$$

$$D_1^{gI} = C_A\left[-\frac{1}{\hat{t}} - \frac{1}{\hat{s}} + 2\frac{m_c^2}{\hat{t}^2}\frac{12m_c^4\hat{s}^3 - 16m_c^2\hat{s}^2\hat{t}\hat{u} + \hat{t}\hat{u}\left(\hat{t}^3 + 3\hat{s}\hat{t}\hat{u} + \hat{u}^3\right)}{\hat{s}\left(-4m_c^4\hat{s}^2 + 4m_c^2\hat{s}\hat{t}\hat{u} + \hat{t}^3\hat{u} + \hat{t}\hat{u}^3\right)}\right]$$

$$D_0^{gI} = C_A\left[-\frac{1}{\hat{t}} - \frac{1}{\hat{s}} + 2\frac{m_c^2}{\hat{t}^2}\frac{24m_c^4\hat{s}^3 - 28m_c^2\hat{s}^2\hat{t}\hat{u} - \hat{s}\hat{t}\hat{u}\left(\hat{t}^2 - 6\hat{t}\hat{u} + \hat{u}^2\right)}{\hat{s}\left(-4m_c^4\hat{s}^2 + 4m_c^2\hat{s}\hat{t}\hat{u} + \hat{t}^3\hat{u} + \hat{t}\hat{u}^3\right)}\right]$$

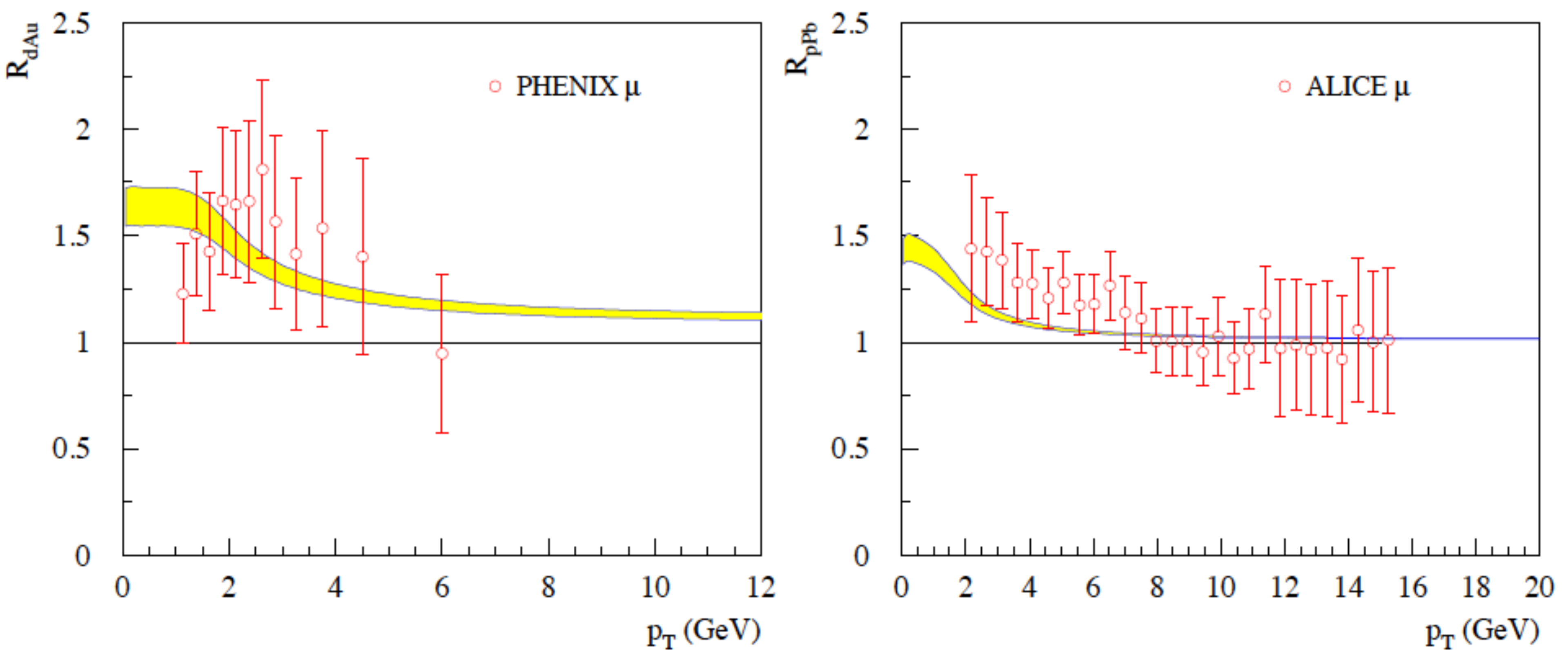
Final

$$D_2^{gF} = C_F\left[-\frac{1}{\hat{t}} - \frac{1}{\hat{u}} - \frac{m_c^2\hat{s}^2}{\hat{t}^2\hat{u}^2}\right]$$

$$D_1^{gF} = C_F\left[-\frac{1}{\hat{t}} - \frac{1}{\hat{u}} + 2\frac{m_c^2\hat{s}}{\hat{t}^2\hat{u}^2}\frac{12m_c^4\hat{s}^3 - 16m_c^2\hat{s}^2\hat{t}\hat{u} + \hat{t}\hat{u}\left(\hat{t}^3 + 3\hat{s}\hat{t}\hat{u} + \hat{u}^3\right)}{-4m_c^4\hat{s}^2 + 4m_c^2\hat{s}\hat{t}\hat{u} + \hat{t}^3\hat{u} + \hat{t}\hat{u}^3}\right]$$

$$D_0^{gF} = C_F\left[-\frac{1}{\hat{t}} - \frac{1}{\hat{u}} + 2\frac{m_c^2\hat{s}}{\hat{t}^2\hat{u}^2}\frac{24m_c^4\hat{s}^3 - 28m_c^2\hat{s}^2\hat{t}\hat{u} - \hat{s}\hat{t}\hat{u}\left(\hat{t}^2 - 6\hat{t}\hat{u} + \hat{u}^2\right)}{-4m_c^4\hat{s}^2 + 4m_c^2\hat{s}\hat{t}\hat{u} + \hat{t}^3\hat{u} + \hat{t}\hat{u}^3}\right]$$

- Nuclear modification factor – single muon decayed from heavy flavor



Incoherent multiple scattering leads to significant enhancement effect in intermediate pt region.

Summary

- Using SIDIS and DY dilepton production, we verified QCD factorization for multiple scattering at one loop order at twist-4.

- We derived the QCD evolution equation for qhat.

- We apply the factorization for multiple scattering to study single inclusive particle production in p+A collisions, including both light and heavy.

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Thanks !