

# E-loss, radiative transport, viscous $\delta f$

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# GLV + bulk medium

DM & Deke Sun

$$\Delta E_{GLV}^{(1)}(z) = \frac{C_R \alpha_s}{\pi^2} \chi \int dx dk dq \frac{\mu^2}{\pi(\mathbf{q}^2 + \mu^2)^2} \frac{2\mathbf{k} \cdot \mathbf{q}}{\mathbf{k}^2(\mathbf{k} - \mathbf{q})^2} (1 - \cos \omega \Delta z)$$

$$E_{GLV} = \int dz \rho \sigma_{gg \rightarrow gg} \Delta E_{GLV}(z)$$

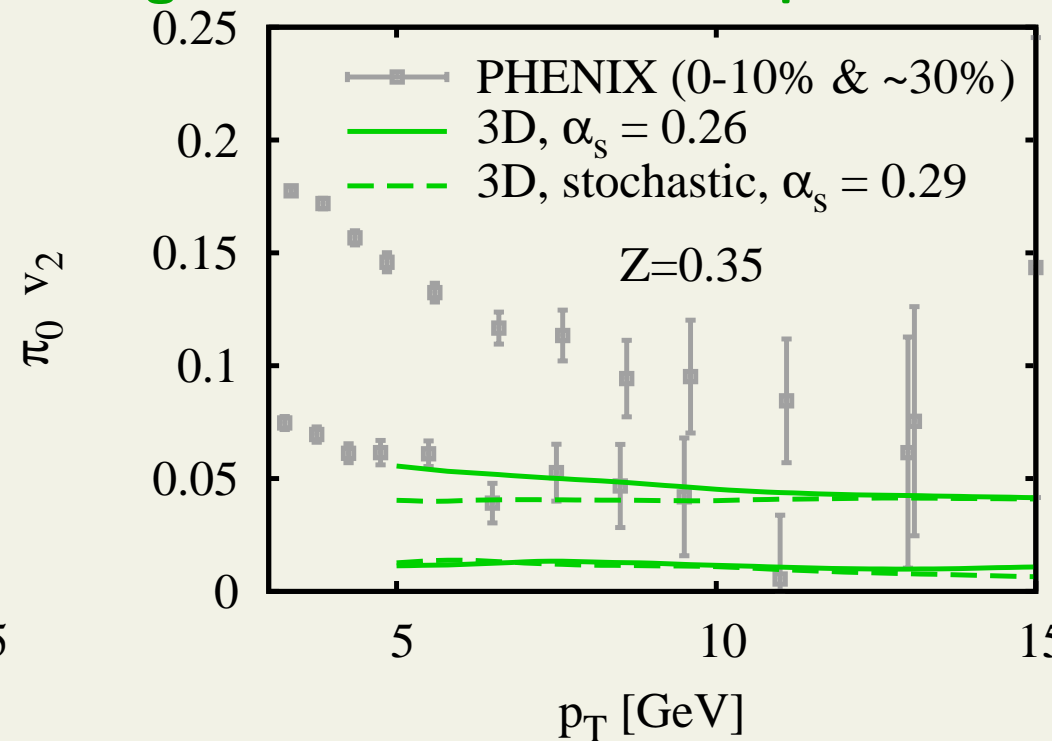
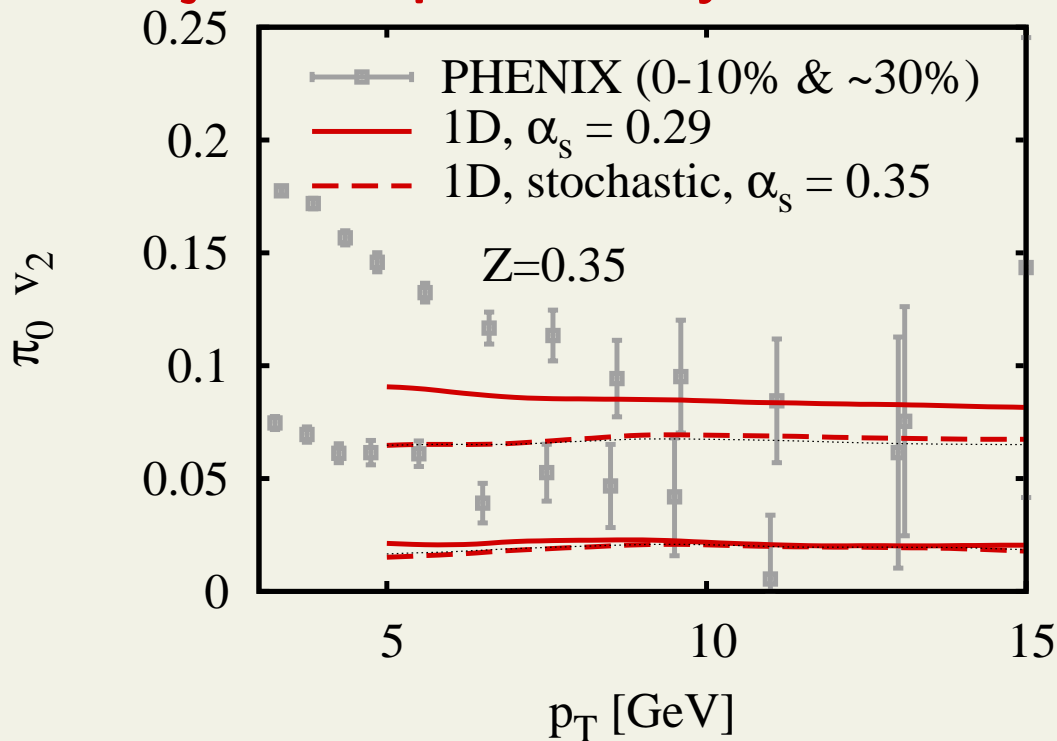
- transverse expansion halves  $v_2$  at RHIC (also LHC)

DM & Sun, NPA901-911 ('13) & 1305.1046

### Bjorken expansion only

vs

### longitudinal + transverse expansion



**BUT claim:**  $R_{AA}(\phi)$  works in 200-GeV at RHIC Betz & Gyulassy, 1305.6458

$$\frac{dE}{dL} = \kappa \frac{C_{jet}}{C_g} E^a L^b T^c(L)$$

with “pQCD-like”  $a = 1/3$ ,  $b = 1$ ,  $c = 2 - a + b = 8/3$

**Apparent contradiction, but calculations differ**

- medium evolution

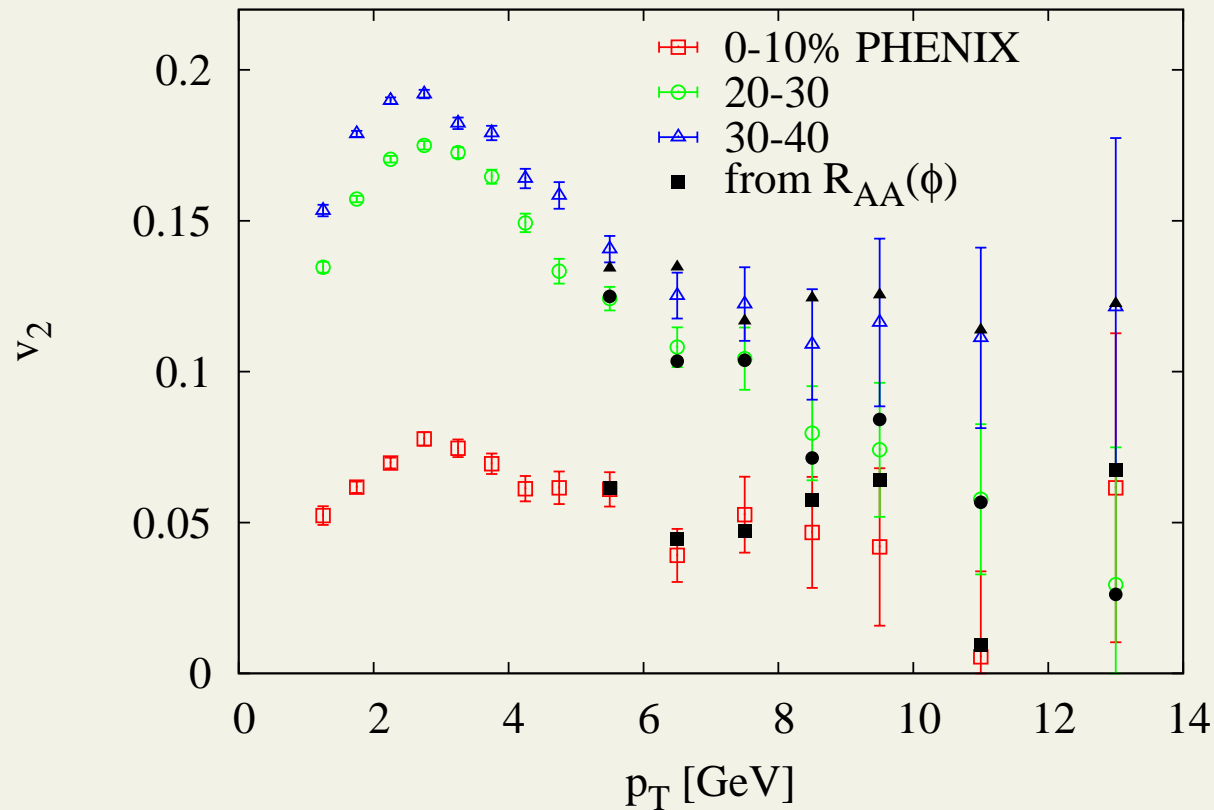
- energy loss model

[- observable studied]

**we resolved this → hydro model/EOS(!)** DM & Sun, arXiv:1405.4848

$$\frac{dN_{AA}}{d^2p_T} = \frac{dN_{AA}}{2\pi p_T dp_T} (1 + 2v_2 \cos 2\phi) \rightarrow R_{AA}(\phi, \Delta\phi) = R_{AA} \left( 1 + 2v_2 \cos 2\phi \frac{\sin \Delta\phi}{\Delta\phi} \right)$$

**pion**  $v_2$  PHENIX PRL 105 ('10) **vs**  $R_{AA}(0, \pi/6)$ ,  $R_{AA}(\pi/2, \pi/6)$  PHENIX PRC87 ('13)



$\Rightarrow$  **datasets are consistent**

## Recheck results with **four** different hydro solutions...

from <https://wiki.bnl.gov/TECHQM>, computed VISH2+1 by OSU group

**“older OSU”** Song, Heinz, Moreland et al arXiv:0709.0742, 0712.3715, 0805.1756

i) ideal hydro, fKLN initial profile - **“ideal fKLN”**

ii)  $\eta/s = 0.08$ , fKLN profile - quite similar to i) in practice

**“newer OSU”** - Shen et al, arXiv:1010.1856; Renk et al, arXiv:1010.1635

iii)  $\eta/s = 0.08$ , Glauber profile - **“visc. Glaub”**

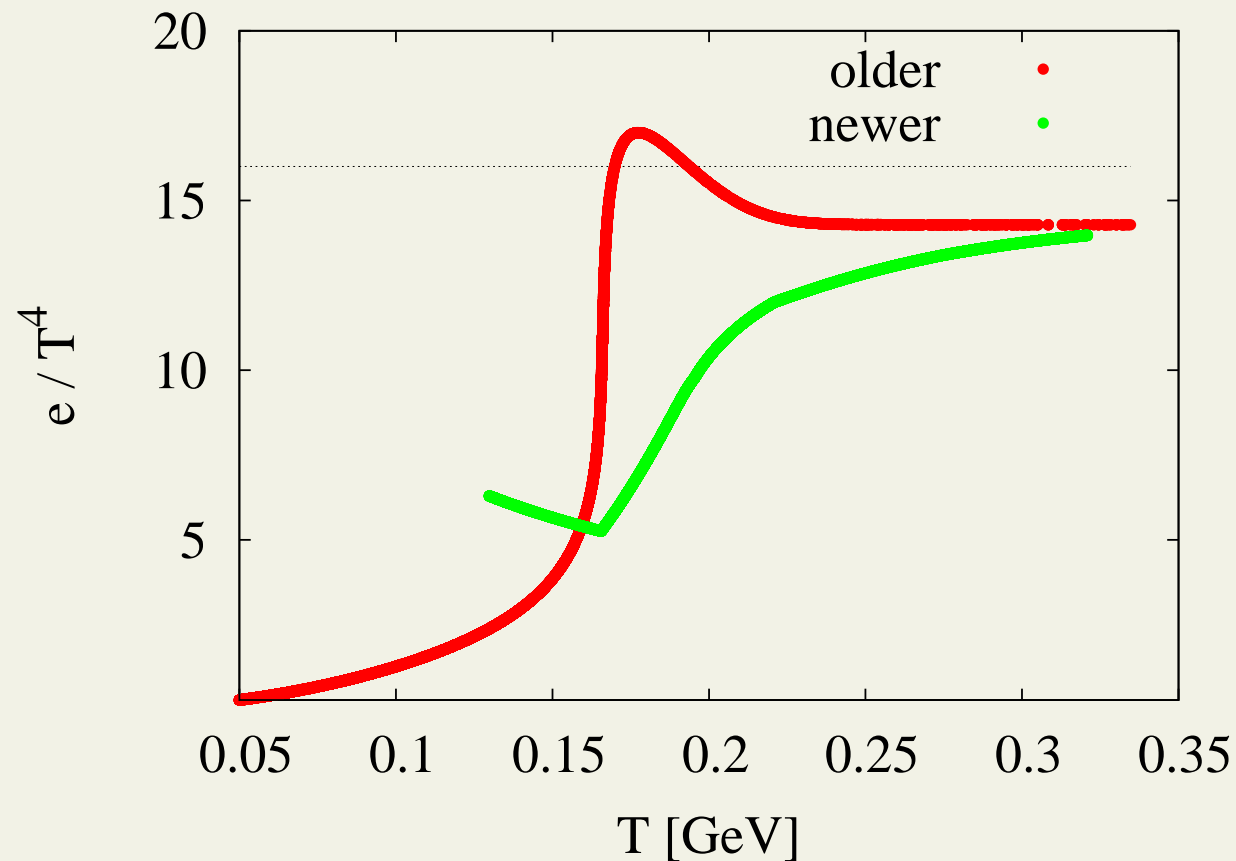
iv)  $\eta/s = 0.08$ , fKLN profile - **“visc. fKLN”**

newer set has most realistic s95-PCE EOS by Huovinen & Petrecky

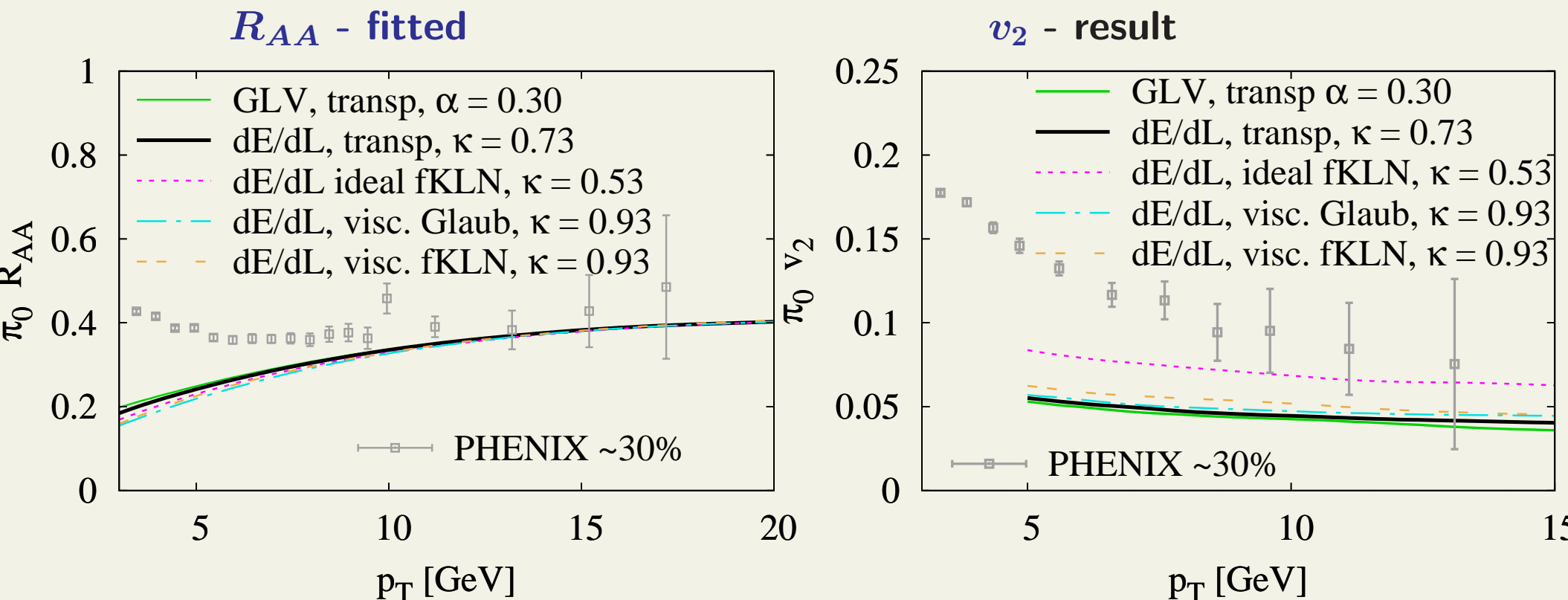
all cases Au+Au at fixed  $b \approx 8$  fm, smooth geometry

**older set:** smoothed Bag Model EoS (SM-EOSQ)

**newer set:** lattice QCD merged onto hadron gas (s95-PCE)



## dE/dL model results for different media DM & Sun ('13)

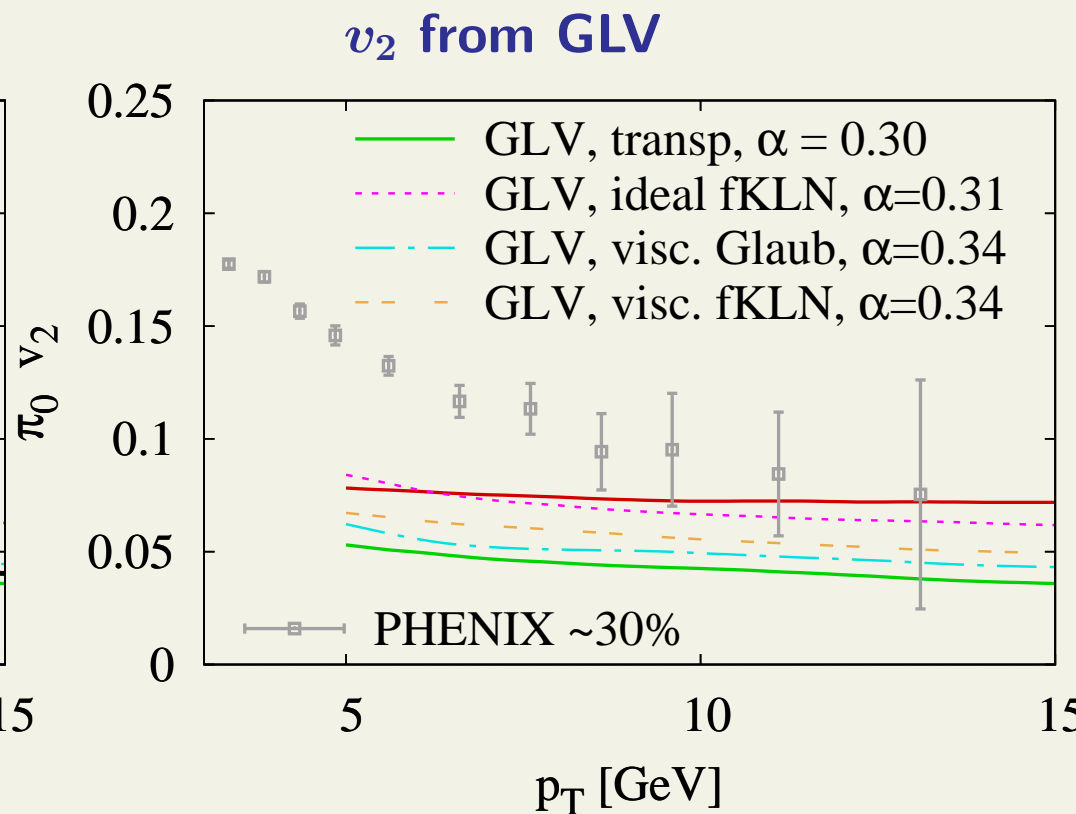
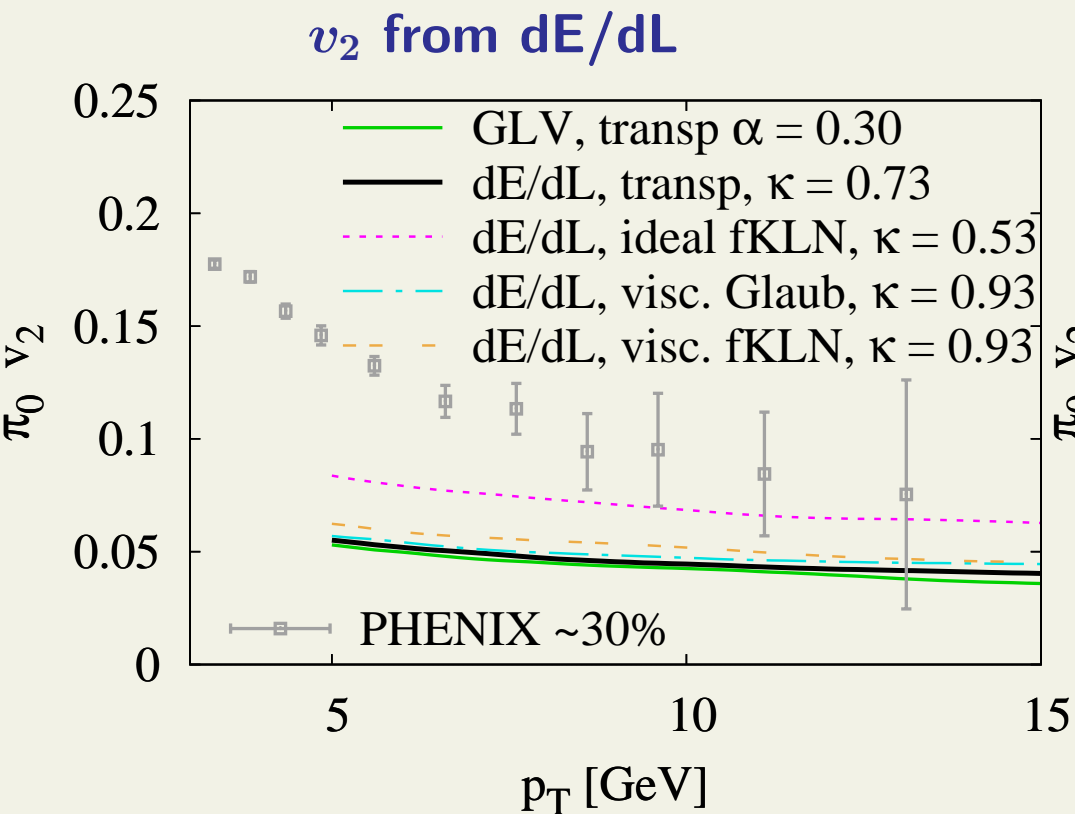


all medium models with dE/dL look very similar to **GLV + transport**

except **“older” OSU hydro calculation** that gives higher  $v_2$



## results for different media DM & Sun, 1405.4848



**GLV and “pQCD-like” dE/dL quite similar**

# Covariance

**Problem:**  $dE/dL = \text{const} \times E^a L^b T^c$  encodes frame dependent physics

Suppose holds in fluid rest frame ( $u_F = (1, \vec{0})$ ), rewrite it in covariant form.  
For massless particles ( $|\vec{v}| = 1$ ):

$$\Delta E_{LR} = \Delta E \gamma_F (1 - \vec{v}\vec{v}_F), \quad \Delta L_{LR} = \Delta L \gamma_F (1 - \vec{v}\vec{v}_F)$$

so covariantly

$$\frac{dE}{dL} = \frac{dE_{LR}}{dL_{LR}} = \kappa [\gamma_F (1 - \vec{v}\vec{v}_F)]^{a+b} E^a L^b T^c$$

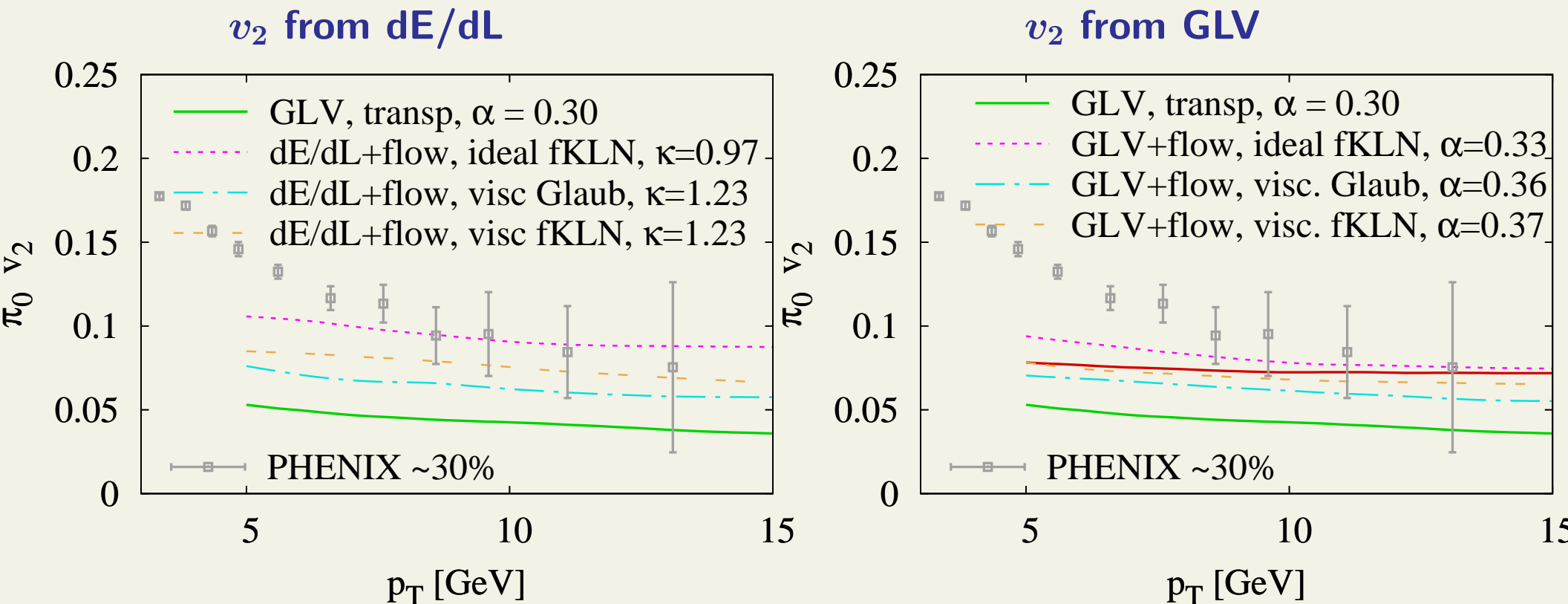
$\Rightarrow$  couples E-loss and medium flow

similar effect argued, e.g., in Baier, Mueller, Schiff nucl-th/0612068v3

Same arises in GLV, in

$$\Delta E = \frac{C_R \alpha_s}{\pi^2} \int dL \rho \sigma_{gg \rightarrow gg} I \left( \frac{\mu^2 L}{E}, \frac{E}{\mu}, \frac{M}{\mu} \right)$$

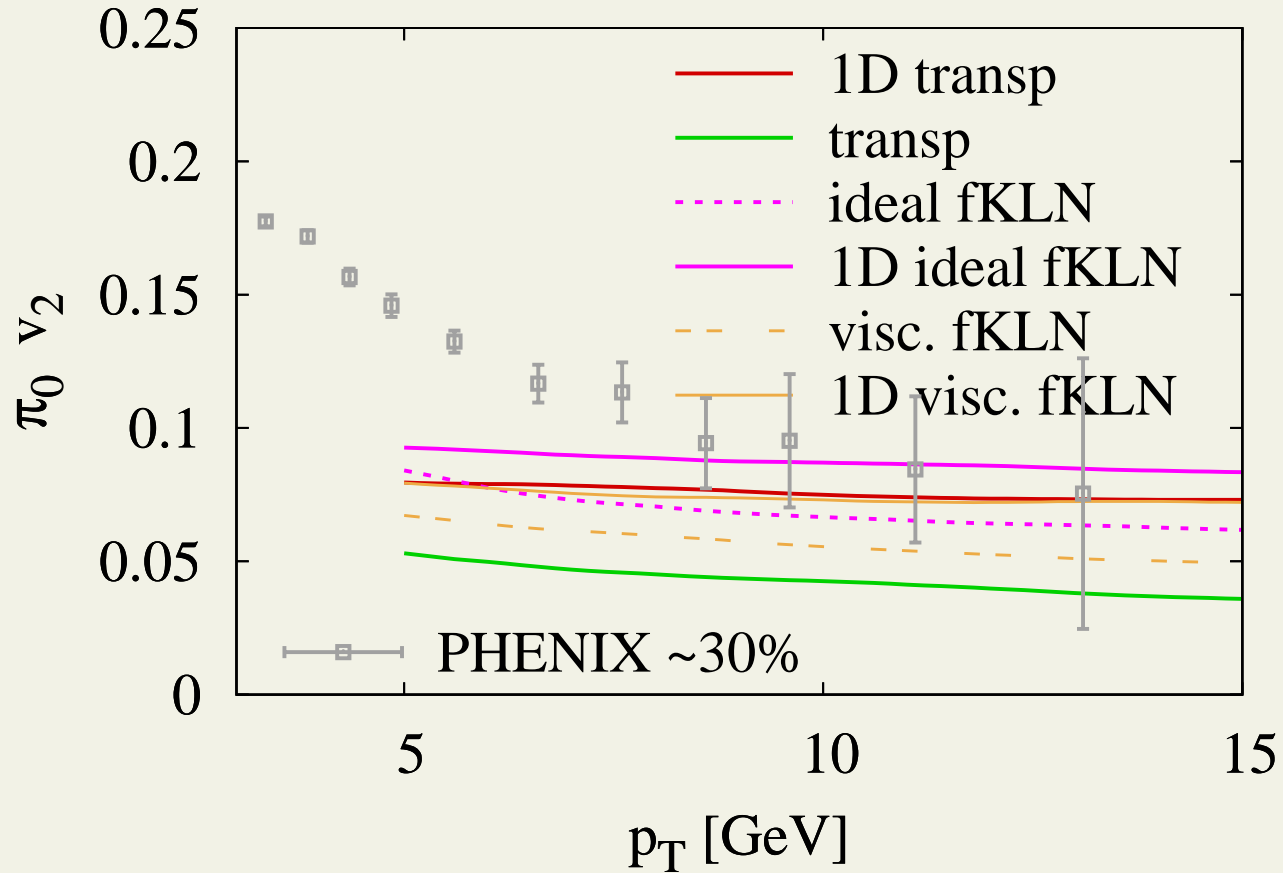
via  $dL \rho \sigma \rightarrow dL_{LR} \rho_{LR} \sigma = dL \rho_{LR} \sigma \gamma_F (1 - \vec{v}\vec{v}_F) = dL \rho \sigma (1 - \vec{v}\vec{v}_F)$



GLV and “pQCD-like” dE/dL quite similar

# 1D vs 3D still matters with naive GLV

DM & Sun ('13)



## Message:

- **nontrivial to get  $R_{AA}$  and  $v_2$  simultaneously**
- **bulk medium background evolution matters**
- **covariance helps**

# Radiative transport MPC/Grid

DM & Dustin Hemphill & Mridula Damodaran

## Radiative transport:

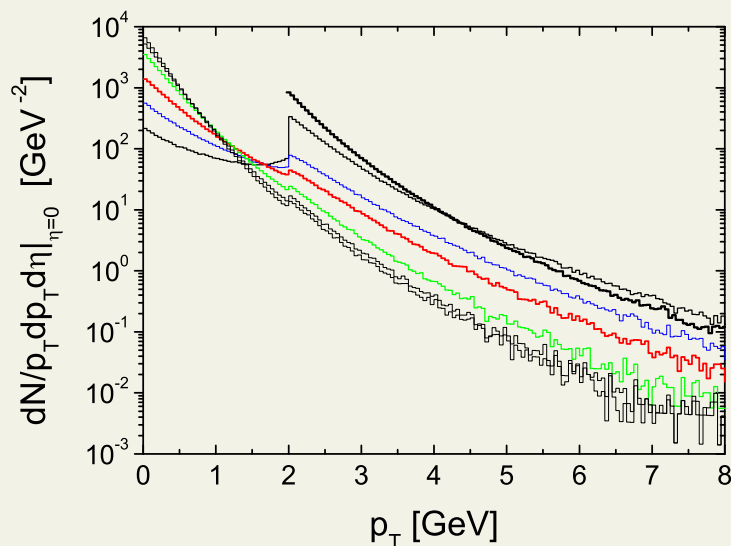
$$p\partial f = S + C_{2\rightarrow 2}[f] + C_{2\leftrightarrow 3}[f] + \dots$$

- Purdue code MPC/Grid for both single-CPU and parallel runs

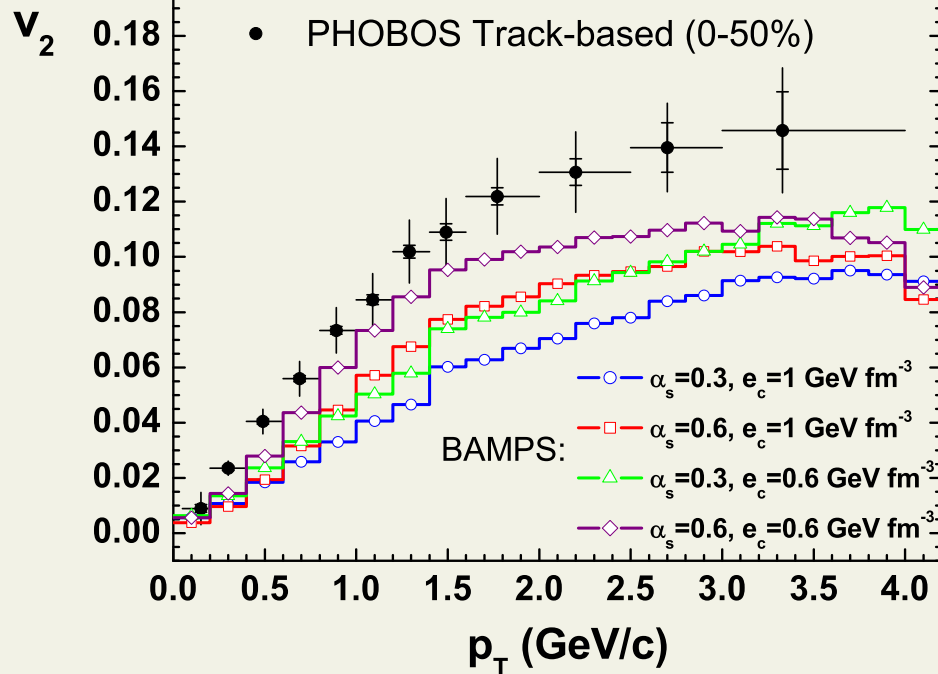
5 useful knobs: cell sizes ( $d_x, d_y, d_z/d_\eta$ ), timestep  $\Delta t$ , subdivision  $\ell$

Big question: does pQCD really thermalize as BAMPS claims?

Xu & Greiner, PRC71 ('05):  $\tau_{eq} \sim 1 fm$



Xu & Greiner, ('08): **significant**  $v_2$



## need QCD rates

$gg \leftrightarrow gg$ :

$$|\bar{\mathcal{M}}|_{LO}^2 = \frac{9g^4}{2} \left( 3 - \frac{us}{t^2} - \frac{ts}{u^2} - \frac{ut}{s^2} \right) \sim \frac{9g^4 s^2}{2} \left( \frac{1}{t^2} + \frac{1}{u^2} \right) \quad (1)$$

**Debye screening:**  $1/t^2 \rightarrow \sim 1/(t - \mu^2)^2$

$gg \leftrightarrow ggg$ :

e.g., Berends et al, PLB 103 ('81)

**12 × 10 terms**

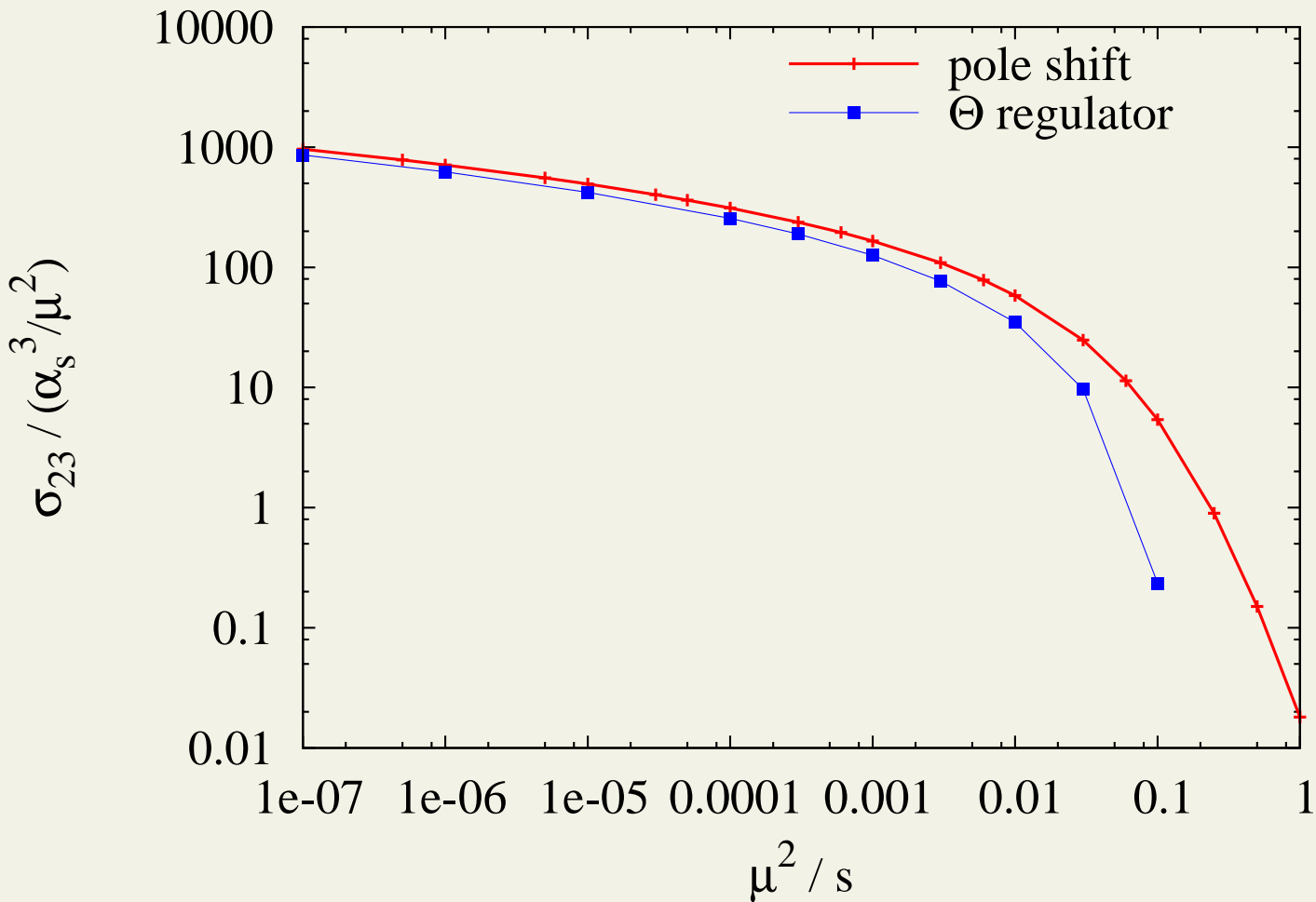
$$|\bar{\mathcal{M}}|_{LO}^2 = \frac{g^6 N_c^3}{2(N_c^2 - 1)} \sum_{perms\{12345\}} \frac{1}{p_{12} p_{23} p_{34} p_{45} p_{51}} \times \sum_{i < j} p_{ij}^4, \quad p_{ij} \equiv p_i p_j \quad (2)$$

**screened by hand:**  $p_{ij} \rightarrow p_i p_j + \mu^2$  Chen et al, PRD83 ('11)

or  $\Theta(p_i p_j - \mu^2)$  Fochler et al



DM & Hemphill ('13) **screened LO**  $gg \rightarrow ggg$



$$\sigma_{2 \rightarrow 2} = \frac{9\pi\alpha_s^2}{2\mu^2}$$

$$\sigma_{2 \rightarrow 3} = \frac{\alpha_s^3}{\mu^2} f\left(\frac{\mu^2}{s}\right)$$

$$\frac{\mu^2}{s} \sim \frac{g^2 T^2}{18T^2} \sim 0.7\alpha_s \sim 0.2 (RHIC)$$

instead of LO pQCD  $gg \leftrightarrow ggg$ :

$$|\bar{\mathcal{M}}|_{LO}^2 = \frac{g^6 N_c^3}{2(N_c^2 - 1)} \sum_{perms\{12345\}} \frac{1}{p_{12} p_{23} p_{34} p_{45} p_{51}} \times \sum_{i<j} p_{ij}^4, \quad p_{ij} \equiv p_i p_j \quad (3)$$

**BAMPS has Bertsch-Gunion form (valid for  $q_T, k_T \ll \sqrt{s}$ ,  $y \approx 0$ )**

$$|\bar{\mathcal{M}}|_{BG}^2 = \frac{54g^6 s^2}{\vec{q}_T^2 \vec{k}_T^2 (\vec{k}_T + \vec{q}_T)^2} \rightarrow \frac{54g^6 s^2 q_T^2 \Theta(k_T \text{ch} y - 1/\lambda_{MFP})}{(q_T^2 + \mu^2)^2 k_T^2 [(\vec{k}_T + \vec{q}_T)^2 + \mu^2]} \quad (4)$$

$$[ p_3 = (k_T e^y, k_T e^{-y}, \vec{k}_T), \quad p_4 = (q_T e^{yi}, q_T e^{-yi}, \vec{q}_T), \quad p_5 = (\sqrt{s}, \sqrt{s}, 0_T) - p_3 - p_4 ]$$

many other choices possible, e.g., fully symmetric Bertsch-Gunion

$$|\bar{\mathcal{M}}|_{BG}^2 = \frac{54g^6 s^2}{(q_T^2 + \mu^2)(k_T^2 + \mu^2)[(\vec{k}_T + \vec{q}_T)^2 + \mu^2]} \quad (5)$$

# Total $\sigma_{gg \rightarrow ggg}$

$$\begin{aligned} \sigma_{23}^{TOT} &= \frac{1}{3!} \frac{1}{64\pi^5 s} \int \frac{d^3 p_3}{2E_3} \frac{d^3 p_4}{2E_4} \frac{d^3 p_5}{2E_5} |M_{12 \rightarrow 345}|^2 \delta^4(p_1 + p_2 - p_3 - p_4 - p_5) \\ &= \frac{1}{3!} \frac{1}{128\pi^4 s} \int_{-\infty}^{\infty} dy \int_0^{\sqrt{s}/(2 \text{ chy})} dk_T \int_0^{2\pi} d\varphi \int_0^{q_T^{max}} dq_T J(k_z, k_T, q_T, \varphi) \sum_{i=1}^2 |M|_{(i)}^2 \end{aligned}$$

clear-cut calculation, of course depends on  $|M_{gg \rightarrow ggg}|$  used

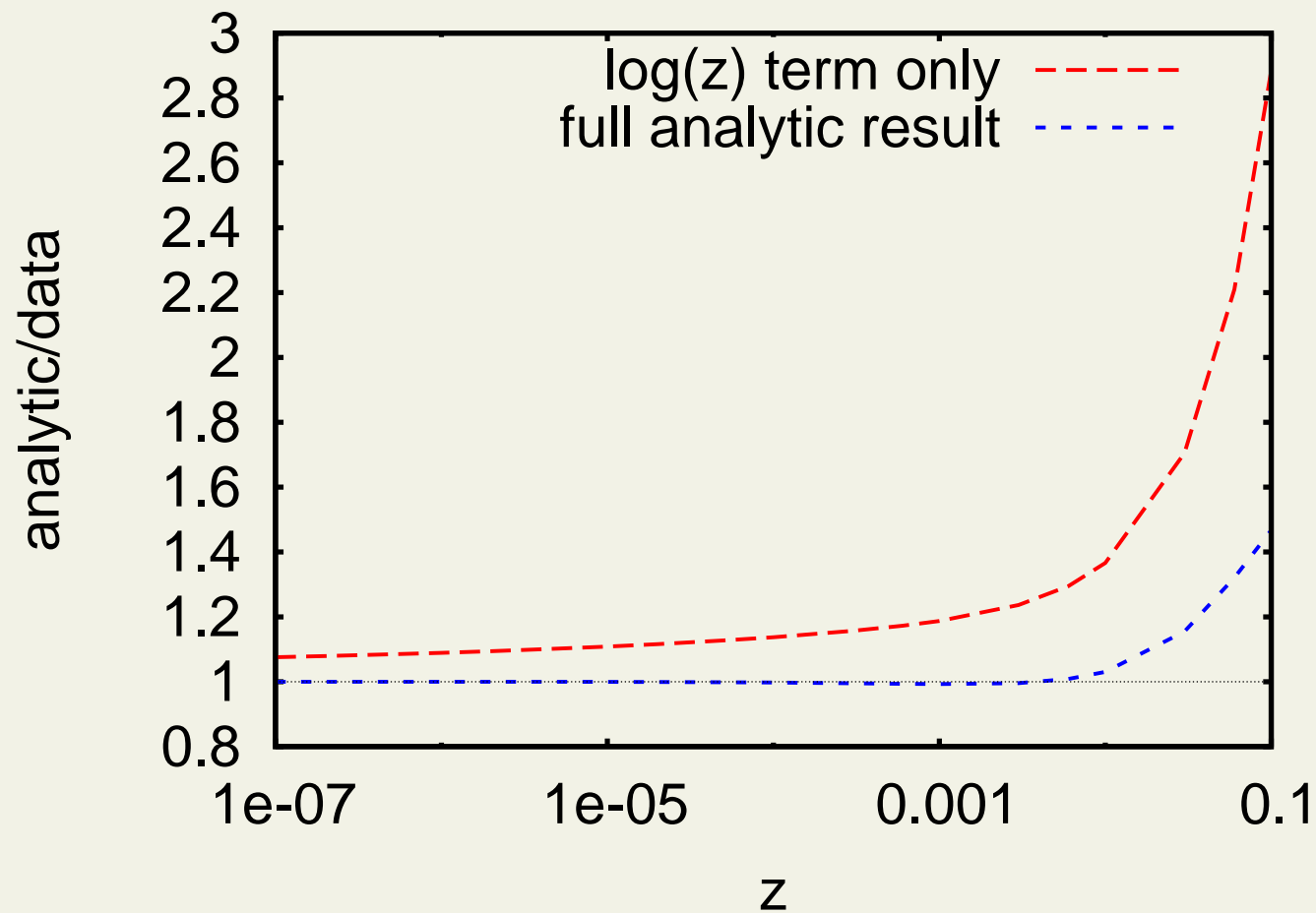
generically, for  $\mu_D^2 \ll s$ :

$$\sigma_{gg \rightarrow ggg} \sim \frac{\alpha_s^3}{\mu_D^2} \ln \frac{s}{\mu_D^2}$$

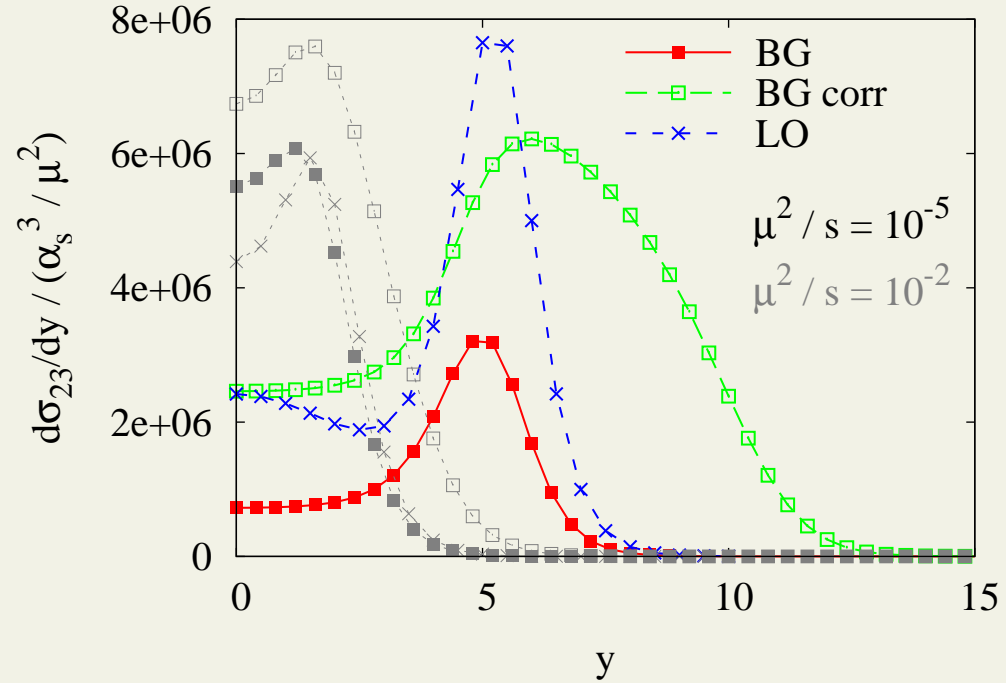
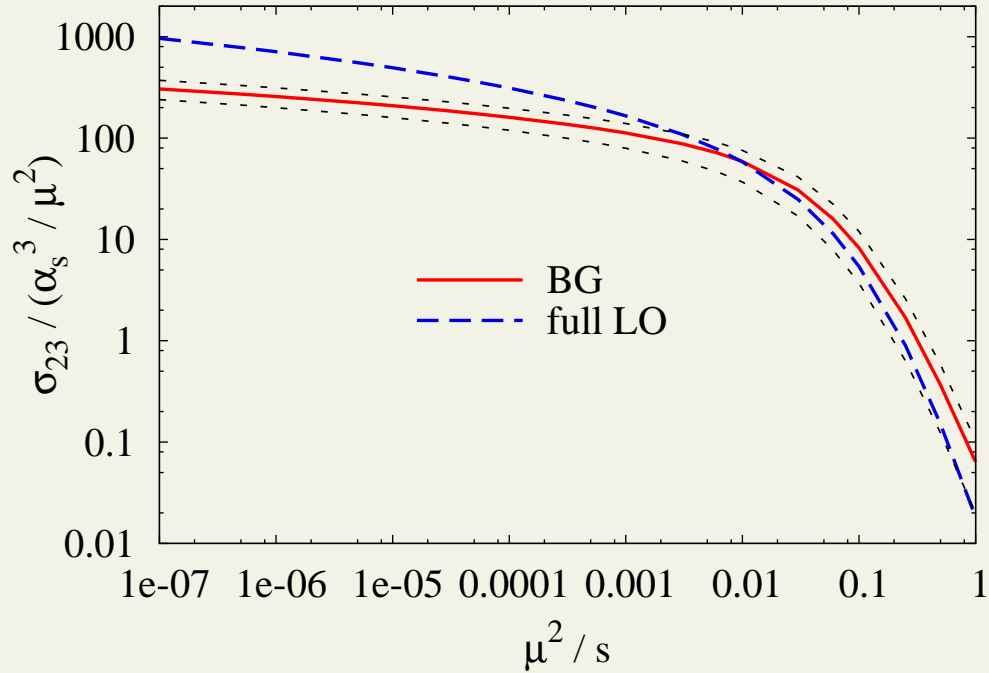
e.g., for fully symmetric **BG**: DM & Hemphill ('14):

$$\sigma_{gg \rightarrow ggg} = \frac{\alpha_s^3}{\mu_D^2} \left[ 31.64 \ln \frac{1}{z} - 35.80 z^0 + \dots \right] \quad (z \equiv \mu_D^2/s)$$

DM & Hemphill ('14): **analytic / numerical  $\sigma_{gg \rightarrow ggg}$  for fully symmetric BG**



## 2 ↔ 3 Bertsch-Gunion (e.g., BAMPS) not so accurate, large rapidities matter



$$|\bar{M}|_{BG}^2 = \frac{54g^6 s^2 \vec{q}_T^2}{(\vec{q}_T^2 + \mu^2)^2 (\vec{k}_T^2 + \mu^2) (\vec{r}_T^2 + \mu^2)}$$

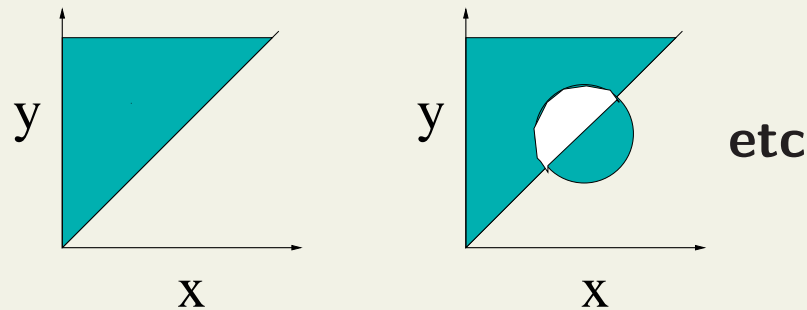
$$|\bar{M}|_{LO}^2 = \frac{g^6 N_c^3}{2(N_c^2 - 1)} \sum_{perms\{12345\}} \frac{1}{p_{12} p_{23} p_{34} p_{45} p_{51}} \times \sum_{i<j} p_{ij}^4, \quad p_{ij} \equiv (p_i p_j)^2 + \mu^2$$

$$\sigma_{23}^{TOT} = \frac{\alpha_s^3}{\mu^2} f\left(\frac{\mu^2}{s}\right)$$

Permutation symmetry can be tricky if  $|M|^2$  is approximated. E.g., in 2D:

$$\frac{1}{2!} \int_0^1 dx \int_0^1 dy |M|^2(x, y) \equiv \int_0^1 dx \int_x^1 dy |M|^2(x, y) \quad \text{if } |M|^2(x, y) = |M|^2(y, x)$$

infinitely many **equivalent** ways to integrate over “half” space



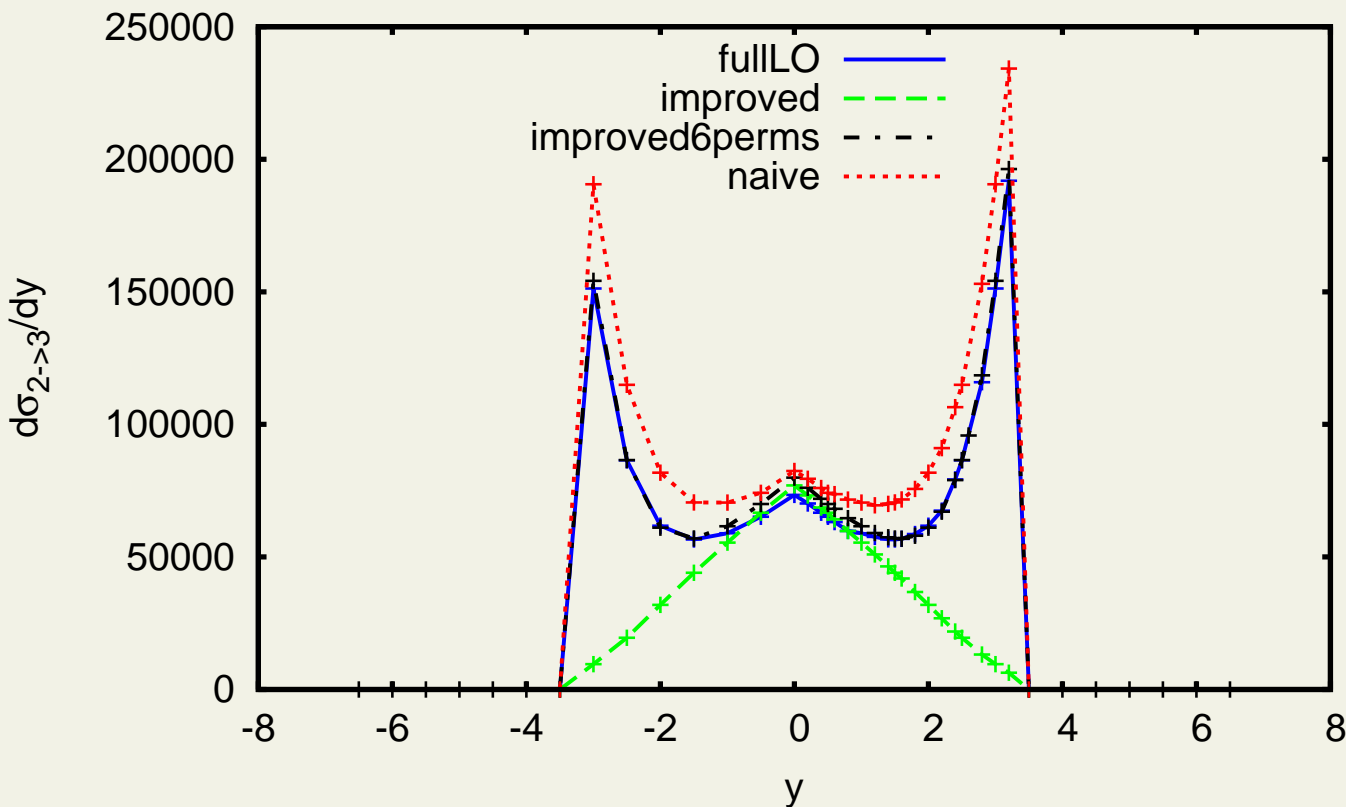
But it does matter which one you take, if your  $|M|^2$  breaks permutation symmetry - like Bertsch-Gunion. However, if approximation is valid over one half, and contributes little over the other half, then  $2!$  may be dropped approximately.

**“improved” Bertsch-Gunion** Fochler et al, 1302.5250

$$|M|_{BG}^2 \rightarrow |M|_{BG}^2 \left( 1 - \frac{p_T}{\sqrt{s}} e^{-|y|} \right)^2$$

$d\sigma_{2 \rightarrow 3}/dy$  vs  $y$

$$\mu_D^2/s = 0.2546, \varepsilon = 0.001, \text{cutoff } \Lambda^2 = \varepsilon * \mu_D^2$$



Hemphill ('13)

$d\sigma_{2 \rightarrow 3}/dy$

**naive BG**

**vs improved BG**

**vs full LO**

**vs permuted improved BG**

sum of all 6 improved BG permutations divided **by 3** works at  $\mu^2/s \approx 0.25$ .

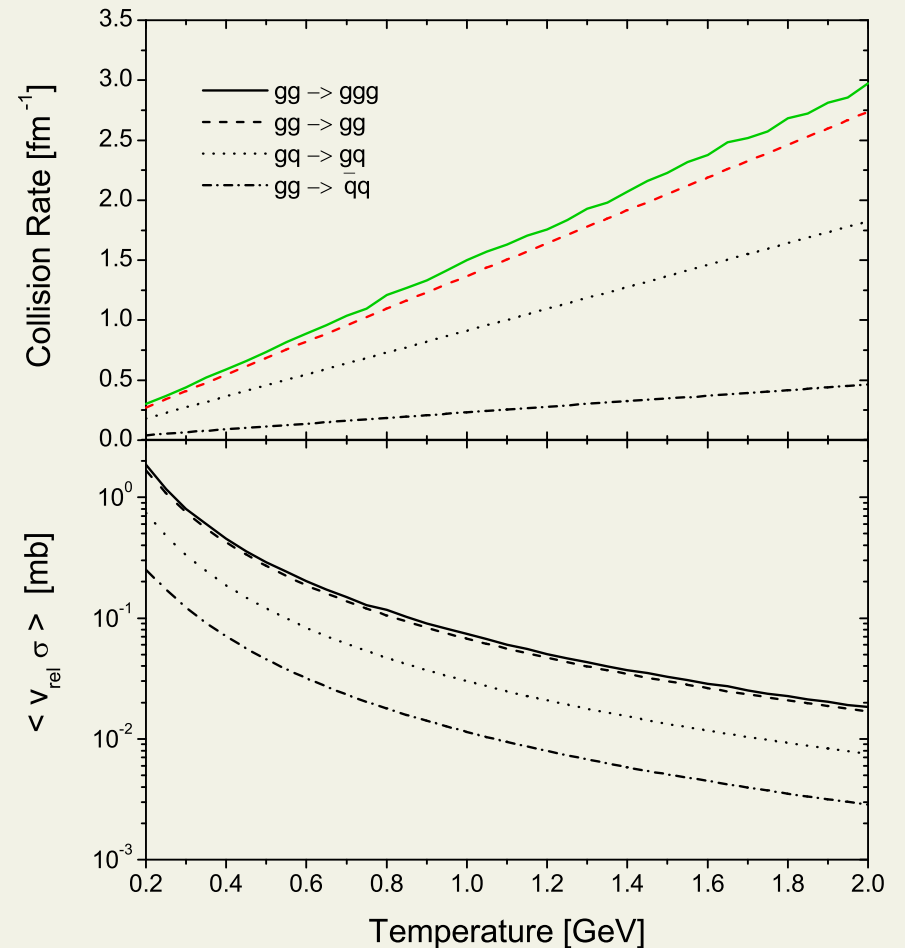
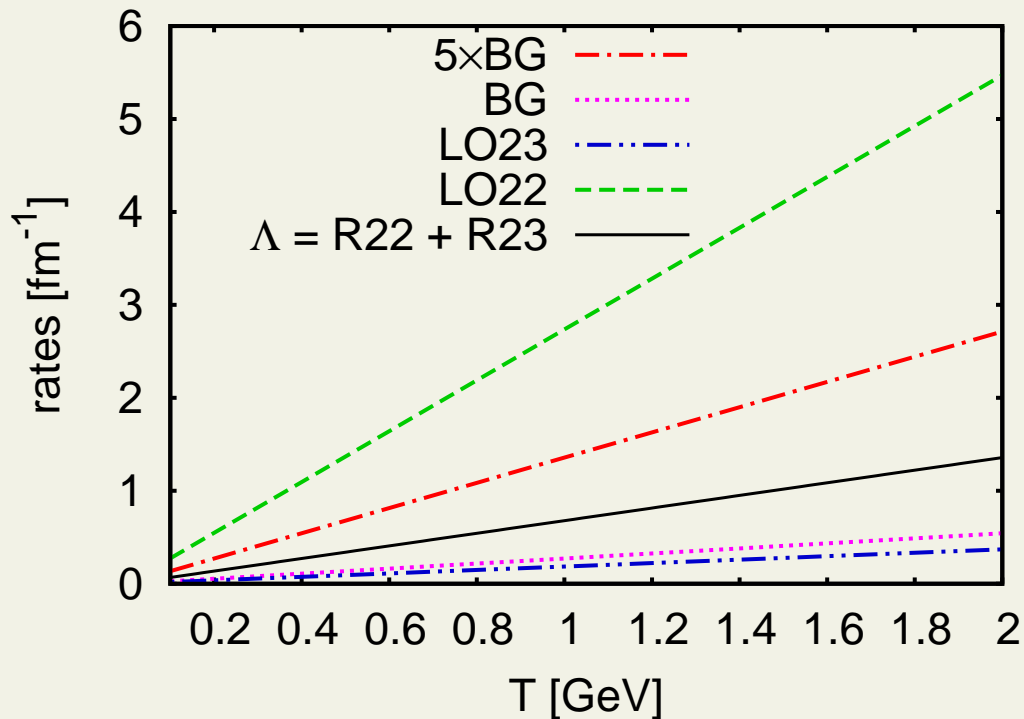
# Thermal rates vs BAMPS

$$\gamma_{2 \rightarrow X} = (2/n) \int d^3p_1 d^3p_2 f_1 f_2 v_{rel} \sigma_{2 \rightarrow X}(s)$$

DM & Hemphill ('13) & ('14)

Xu & Greiner, PRC71 ('05)

smooth  $\mu_D^2$  or LPM cutoff  $\Lambda = 1/MFP$





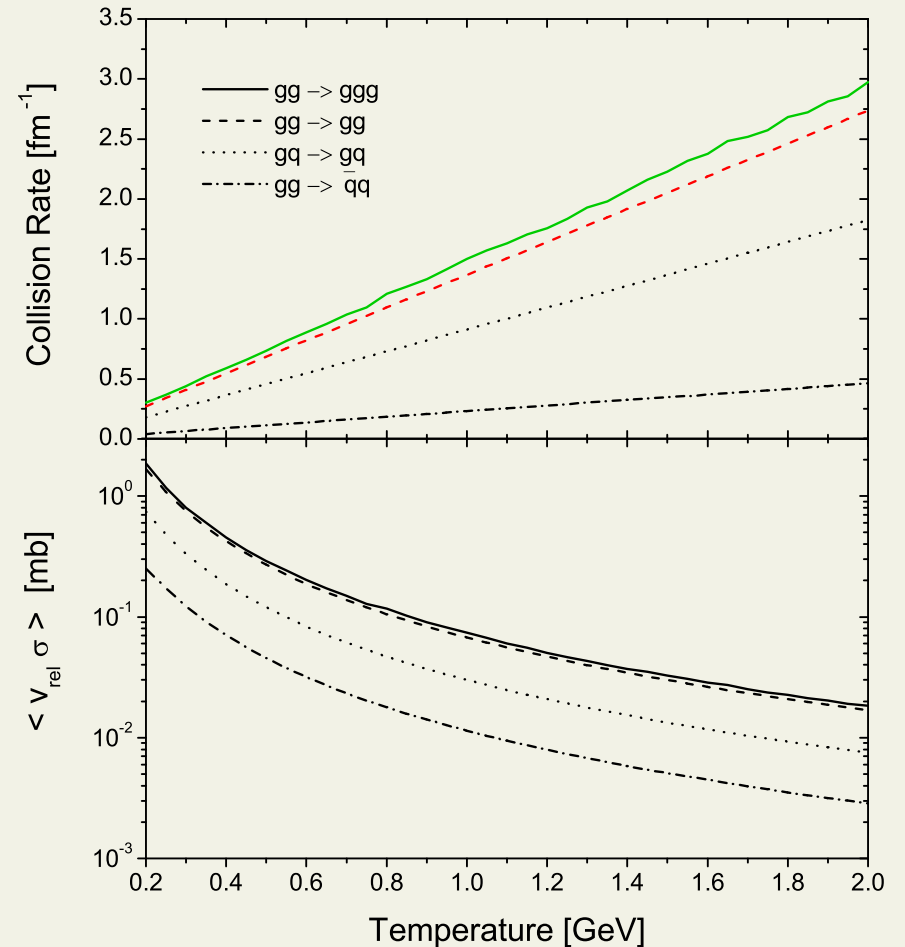
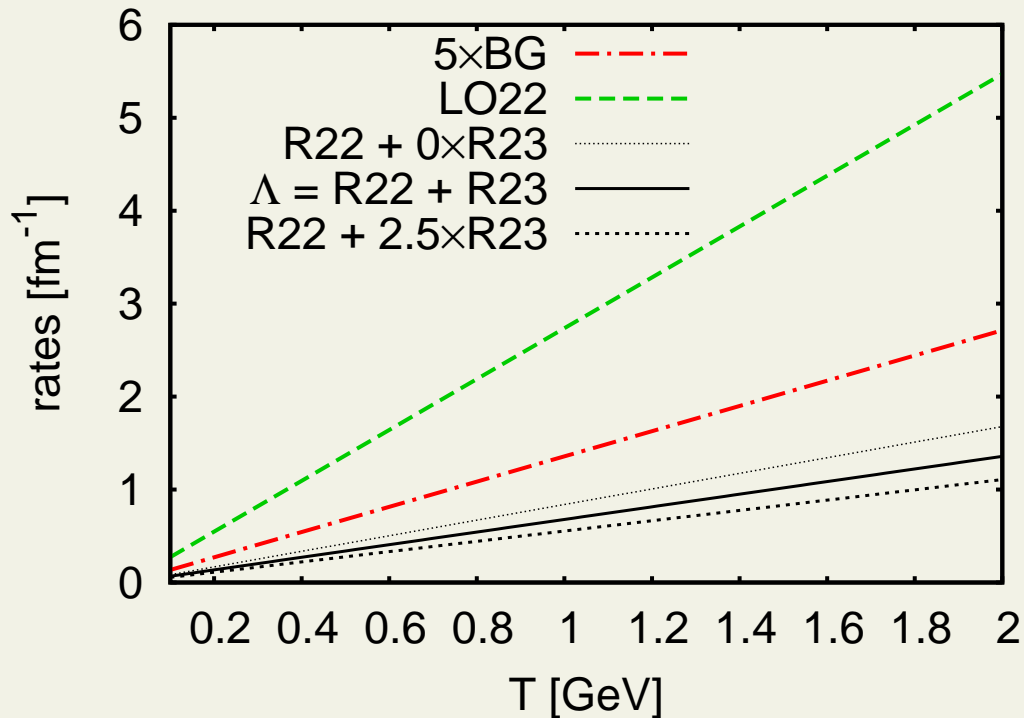
# Thermal rates vs BAMPS

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DM & Hemphill ('13) & ('14)

Xu & Greiner, PRC71 ('05)

## LPM cutoff $\Lambda = 1/MFP$ dependence



# Equilibration in static box

- density estimate for central Au+Au (Glauber binary coll) at  $\tau_0 = 0.6$  fm

$$n = \frac{1}{\tau_0} \frac{dN}{d\eta d^2x_T} = \frac{1}{\tau_0} \frac{dN}{dy} \frac{T_A(\vec{x}_T) T_A(\vec{x}_T)}{T_{AB}(b=0)} \sim 25 \text{ fm}^{-3} \text{ for } x_T = 0$$

[with thickness function  $T_A(\vec{x}) = \int dz \rho_A(\sqrt{x^2 + z^2})$  ]

- momentum distribution: pQCD minijets  $p_T > p_0$ , with  $y = 0$

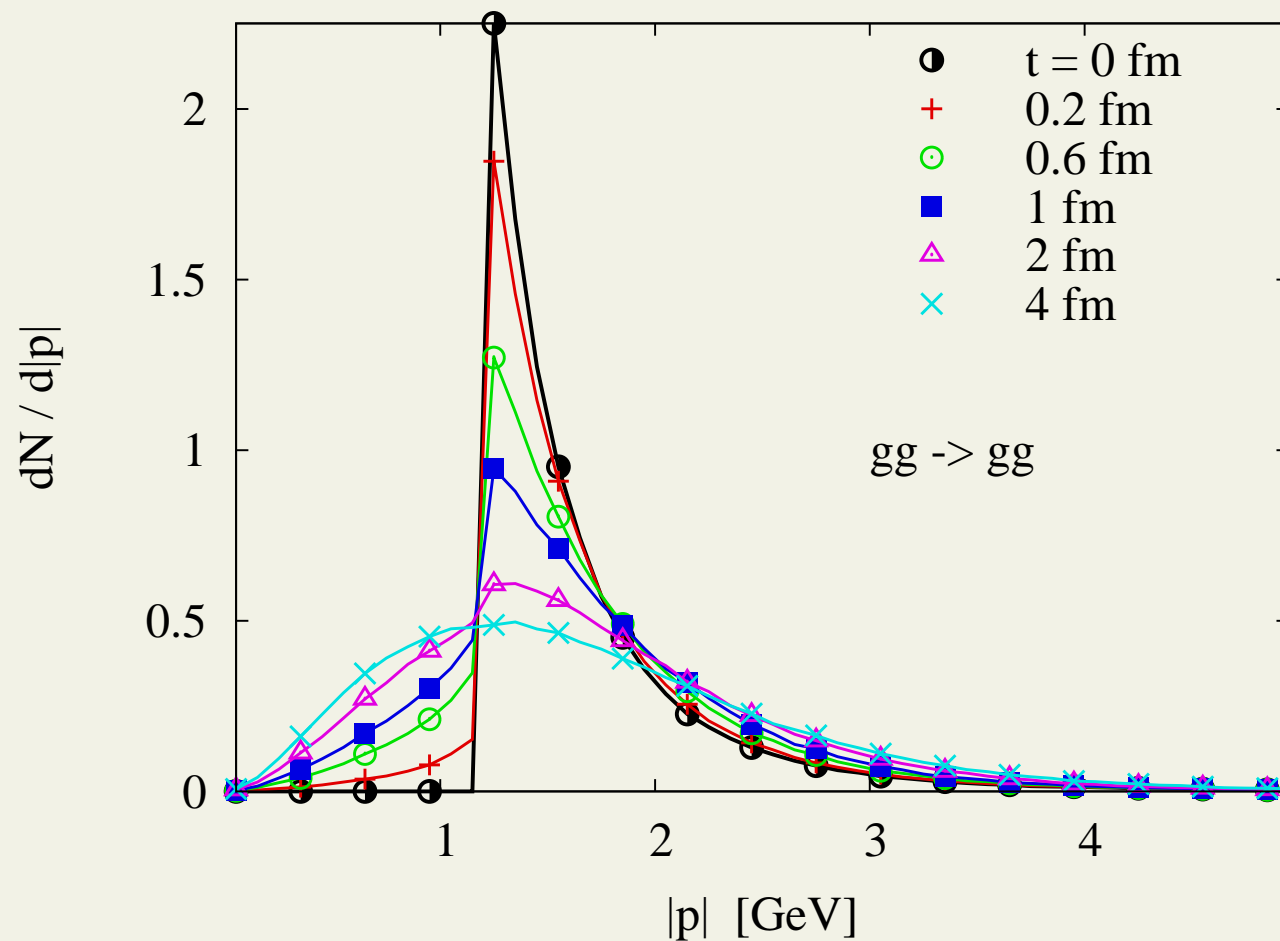
$$\frac{dN}{dy} \approx 1100 \quad \rightarrow \quad p_0 = 1.2 \text{ GeV}$$

(‘a la’ old saturation model Eskola, Kajantie ('98))

- LO  $\sigma_{23}$  with fixed  $\mu_D = gT_{fin} = gE_{TOT}/3N_{fin}$ ,  $\alpha_s = 0.4$

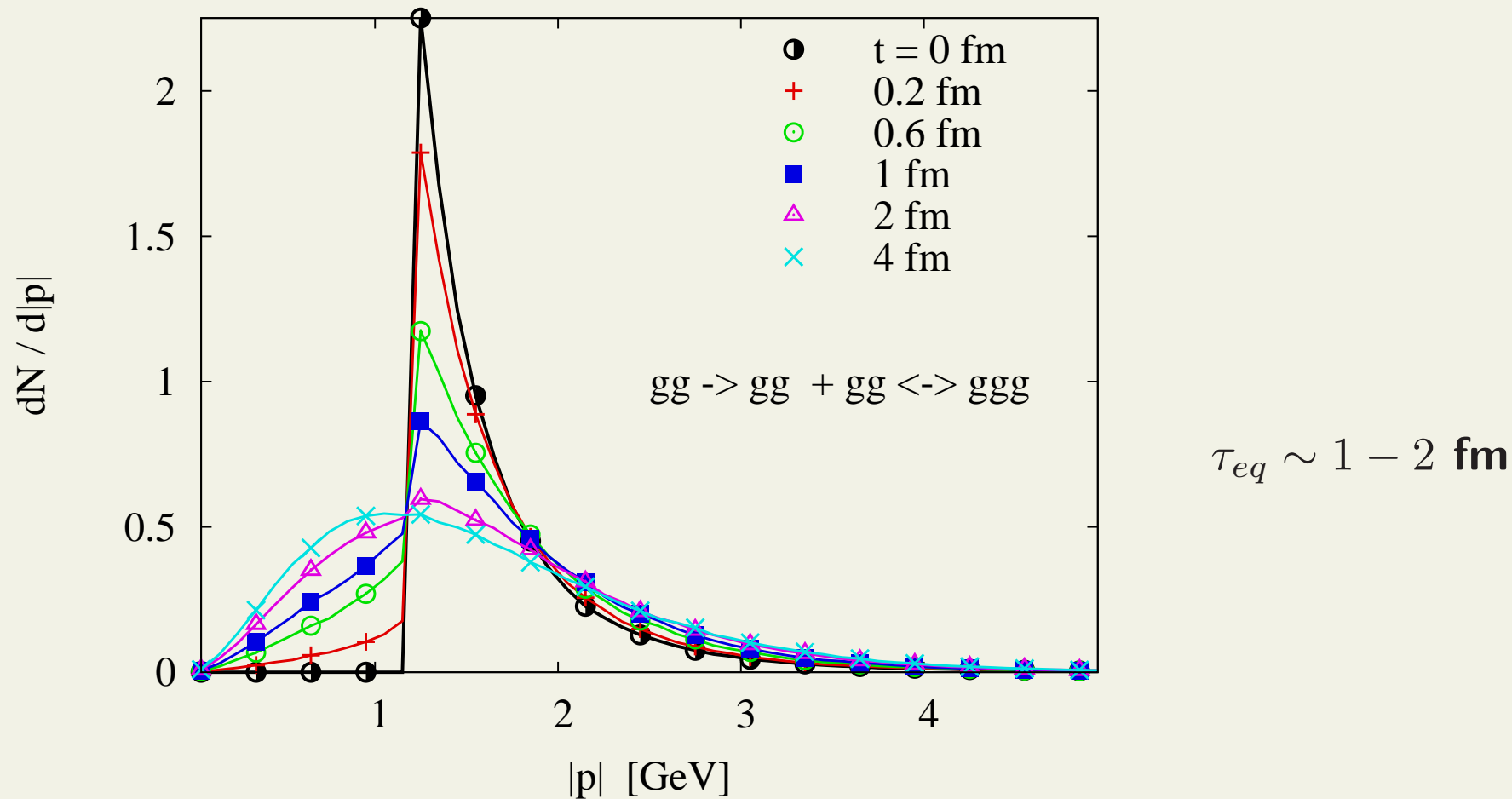
**for simplicity:** use correct  $\sigma_{23}(s)$  but isotropic outgoing momenta

DM & Hemphill ('13): **energy distribution in box** ( $\propto p^2 e^{-p/T}$  in equilibrium)

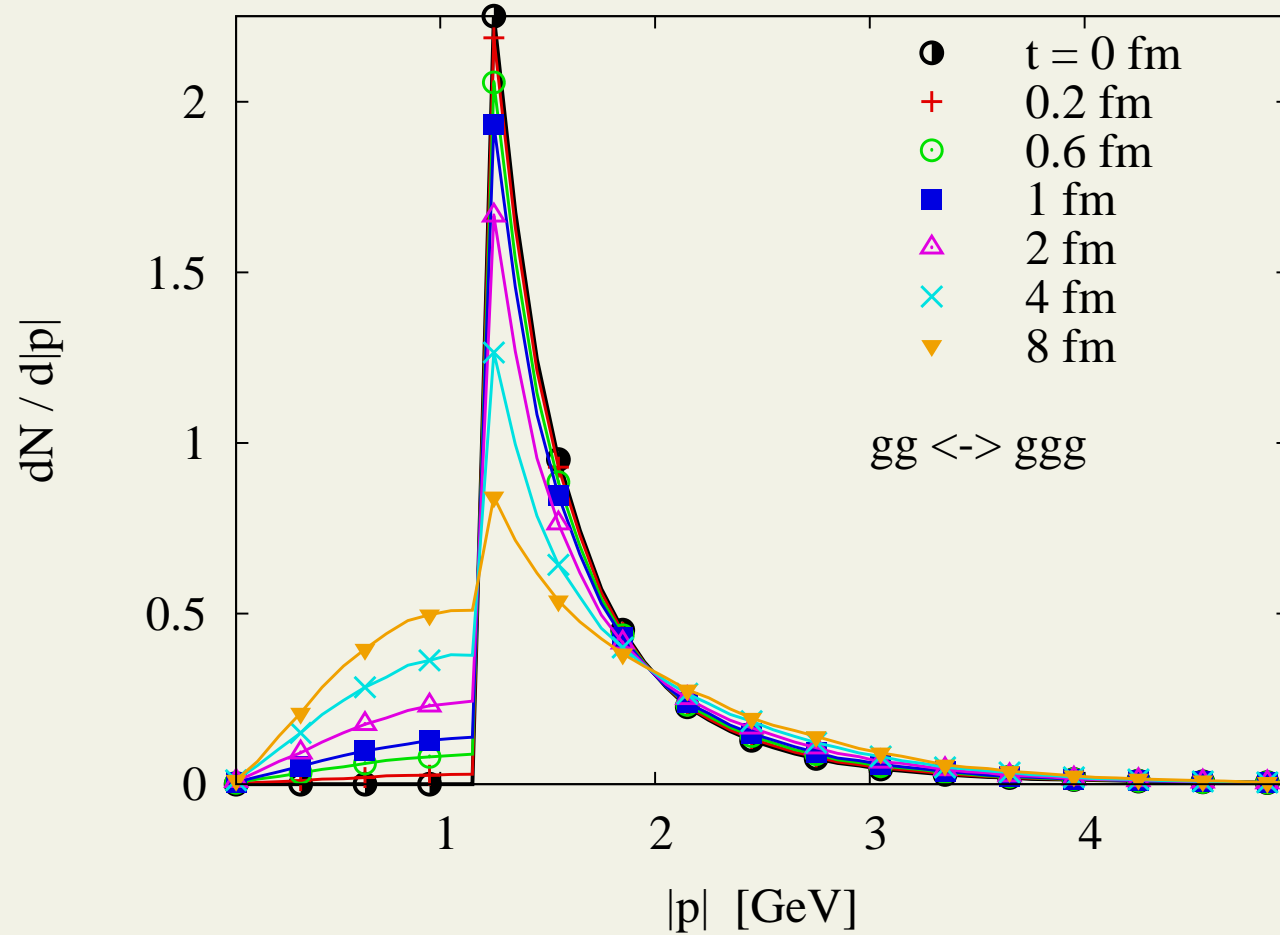


$\tau_{eq} \sim 1 - 2$  fm

DM & Hemphill ('13): **energy distribution in box** ( $\propto p^2 e^{-p/T}$  in equilibrium)



DM & Hemphill ('13): **energy distribution in box** ( $\propto p^2 e^{-p/T}$  in equilibrium)



4 – 8 fm(??)

but **BAMPS** has **local Debye mass** from linear response e.g., Biro et al, PRC48

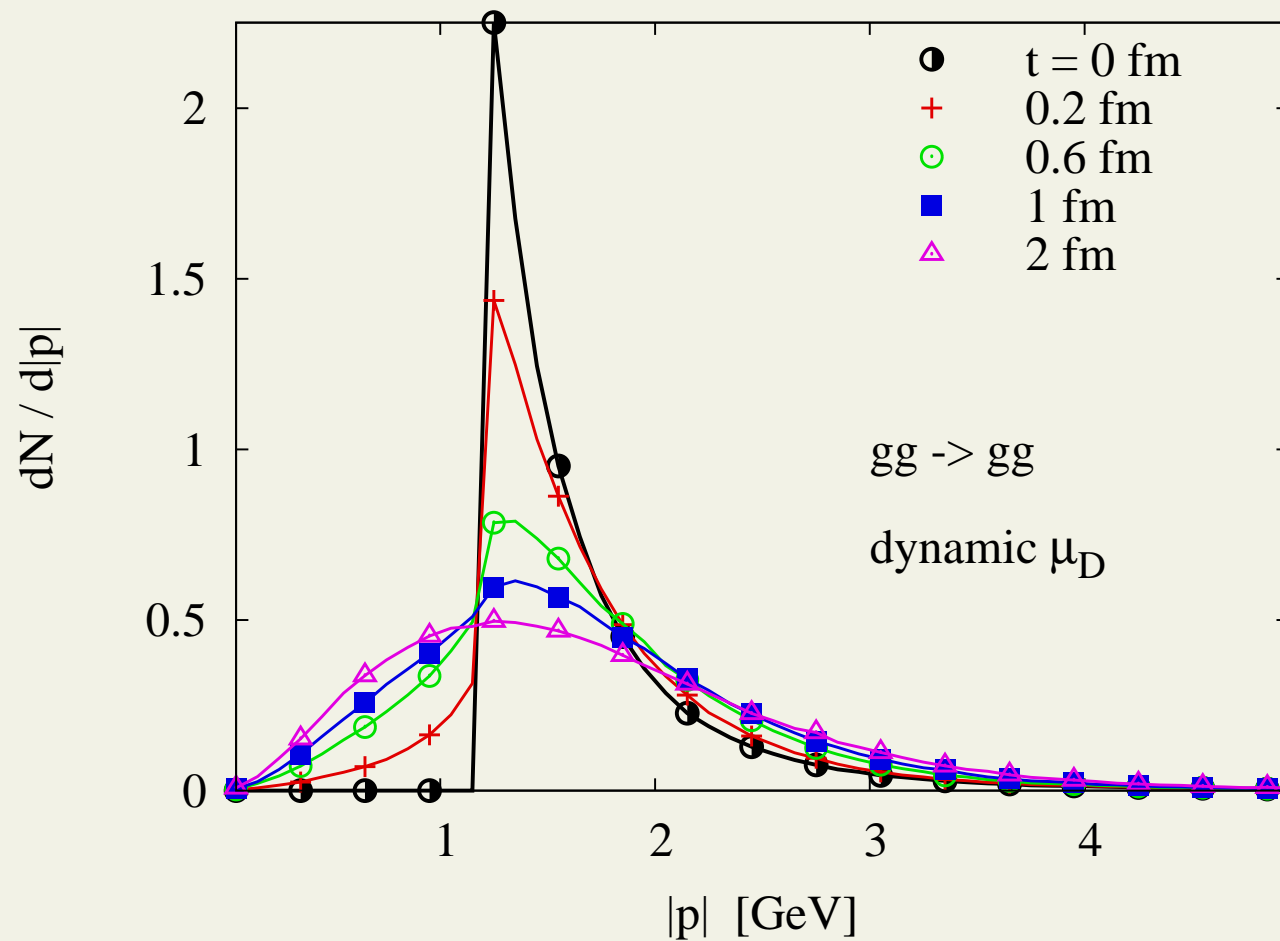
$$\mu_D^2[f] = 3\pi\alpha_s \int d^3p \frac{1}{p} f_g(\vec{p}) \quad [f_{eq} = 16/(2\pi)^3 \times e^{-p/T}]$$

could matter out of equilibrium, so put it in (transverse  $\vec{x}_T$  average for statistics)

Watch out, this reproduces  $\mu_D = gT$  with Bose distribution, but for Boltzmann:

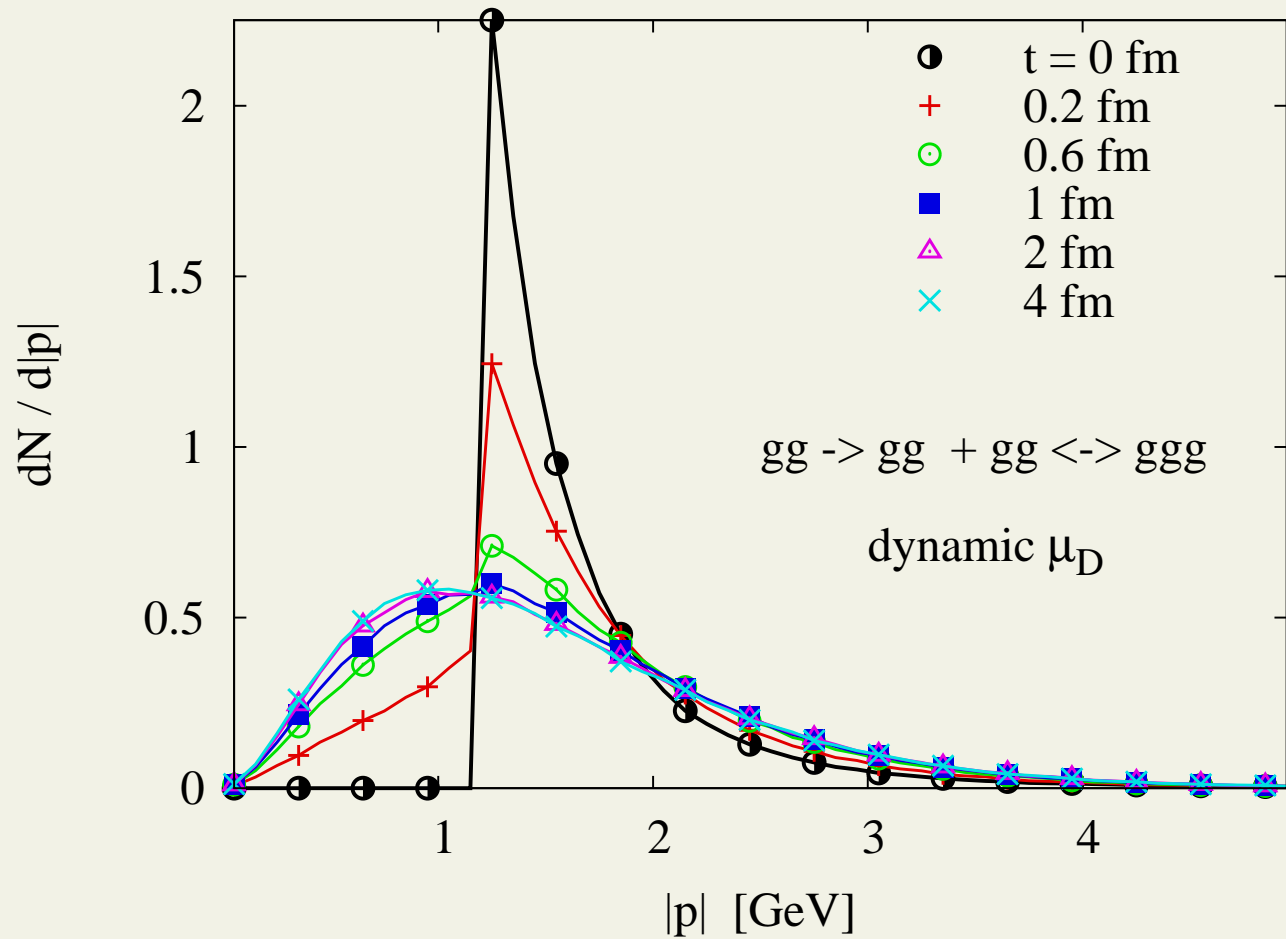
$$\mu_D^2 = \frac{6}{\pi^2} g^2 T^2 < g^2 T^2 \Rightarrow \text{larger rates(!)}$$

DM & Hemphill ('13): **energy distribution in box** ( $\propto p^2 e^{-p/T}$  in equilibrium)



$\tau_{eq} \sim 0.5 - 1$  fm

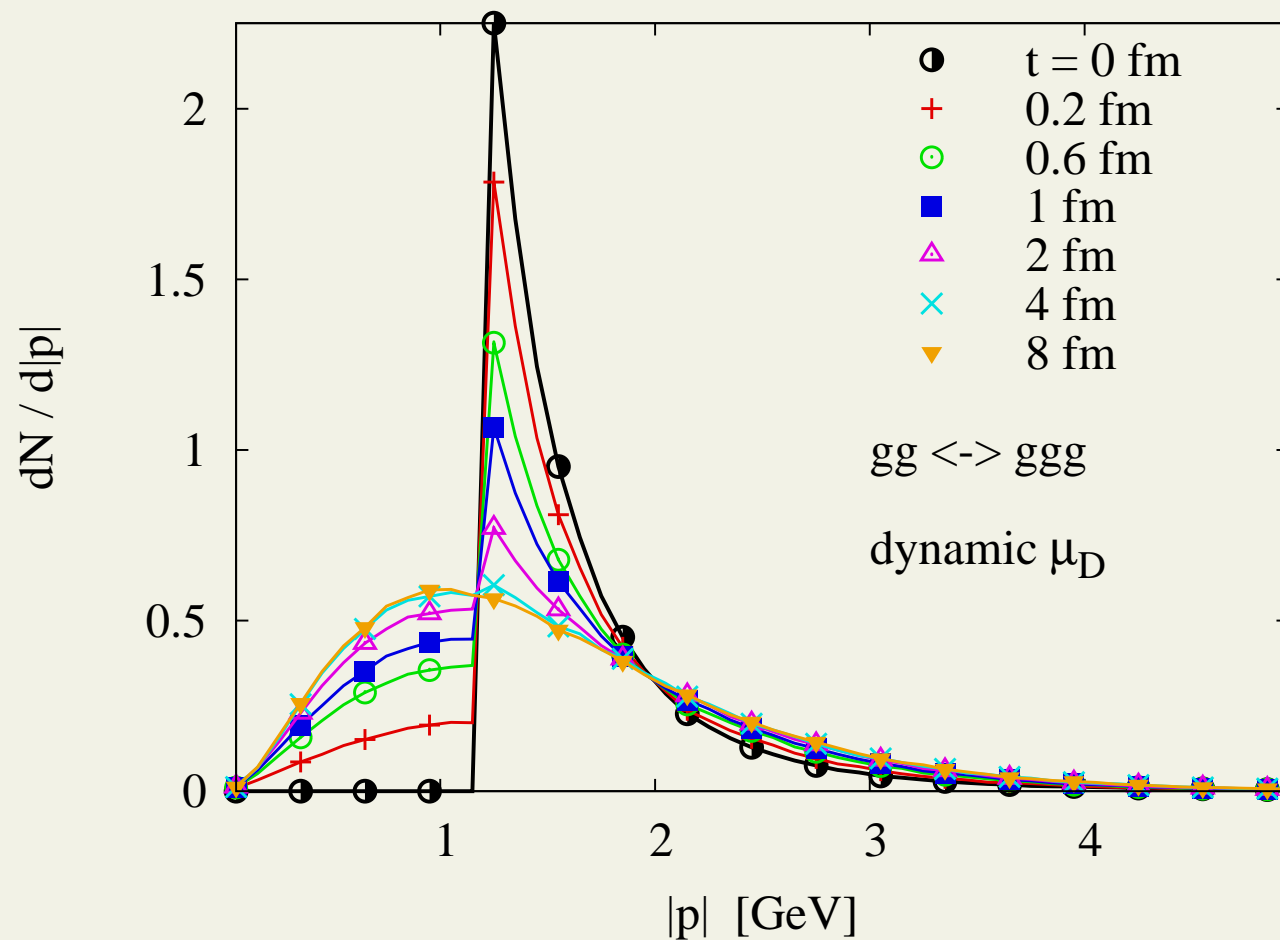
DM & Hemphill ('13): **energy distribution in box** ( $\propto p^2 e^{-p/T}$  in equilibrium)



$\tau_{eq} \sim 0.3 - 0.5$  fm



DM & Hemphill ('13): **energy distribution in box** ( $\propto p^2 e^{-p/T}$  in equilibrium)



1 – 1.5 fm(!)

$6\times$  higher rate due to smaller  $\mu_D$  because  $\sigma_{gg \rightarrow ggg}$  is very sensitive to  $\mu_D$

# Elliptic flow

- longitudinal boost Bjorken invariance

- $\tau_0 = 0.6$  fm

- transverse density: Glauber (binary coll) profile

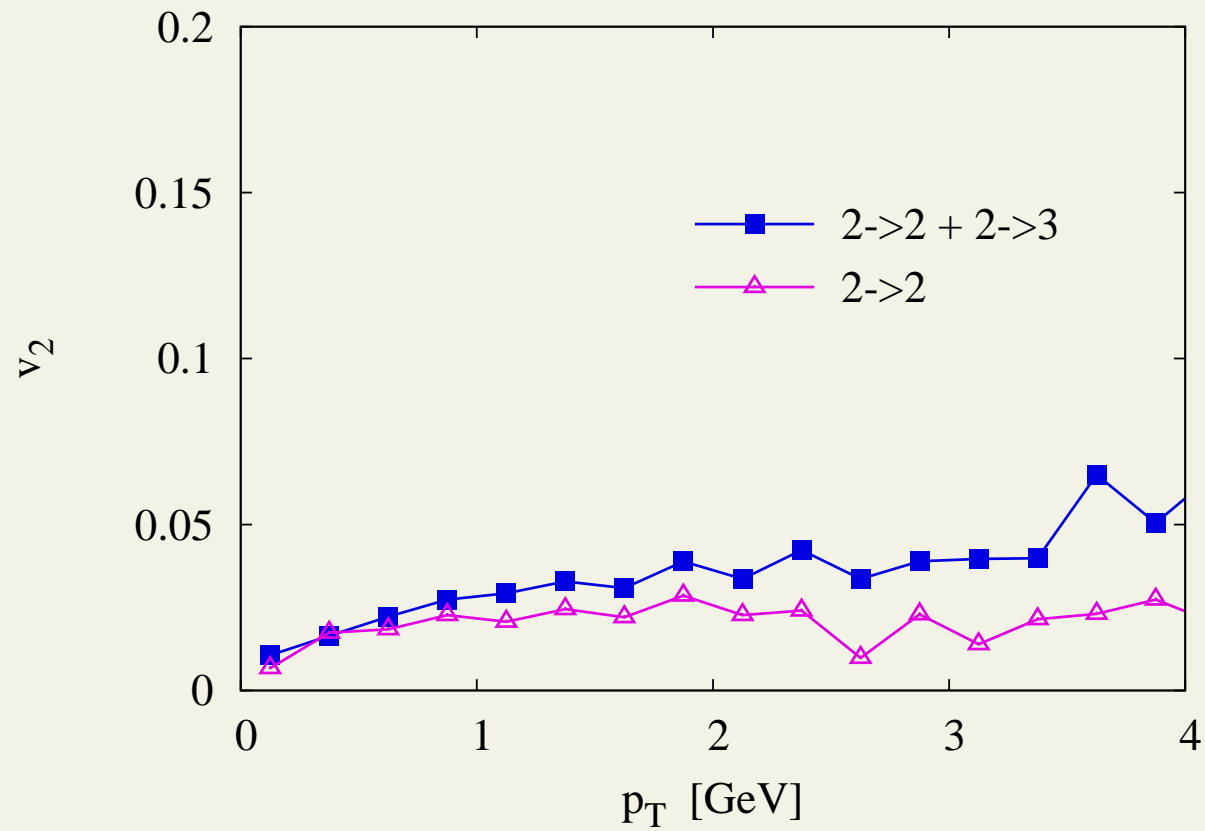
$$\frac{dN}{d\eta d^2x_T} = \frac{dN}{d\eta} \times \frac{T_A(\vec{x}_T - \vec{b}/2)T_A(\vec{x}_T + \vec{b}/2)}{T_{AB}(b)}$$

- momentum distribution: locally thermal at  $T_0 = 0.5$  GeV

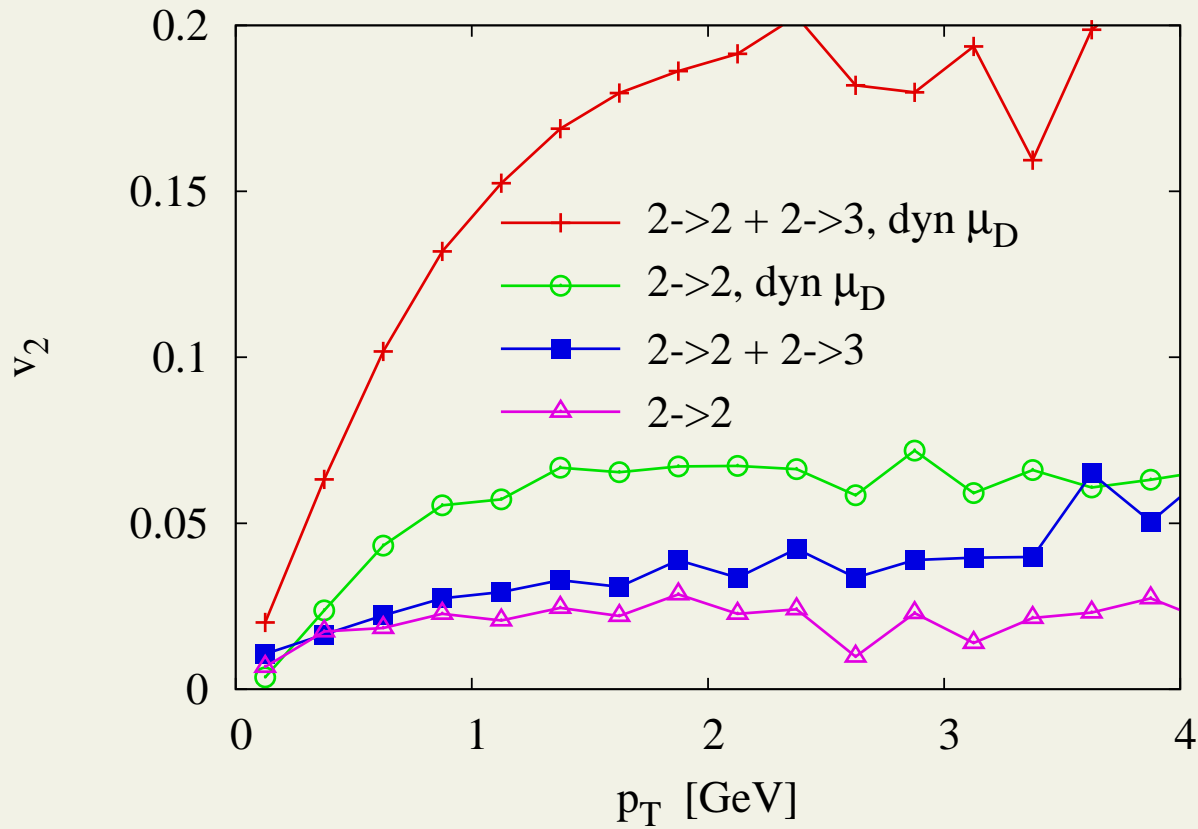
- fixed  $\alpha_s = 0.4$ , and dynamic  $\mu_D[f]$  or  $\mu_D \propto \tau^{-1/3}$

apply to RHIC Au+Au at  $b = 8$  fm

**RHIC Au+Au at  $b = 8$  fm, with  $\mu_D = gT_0(\tau_0/\tau)^{1/3}$**

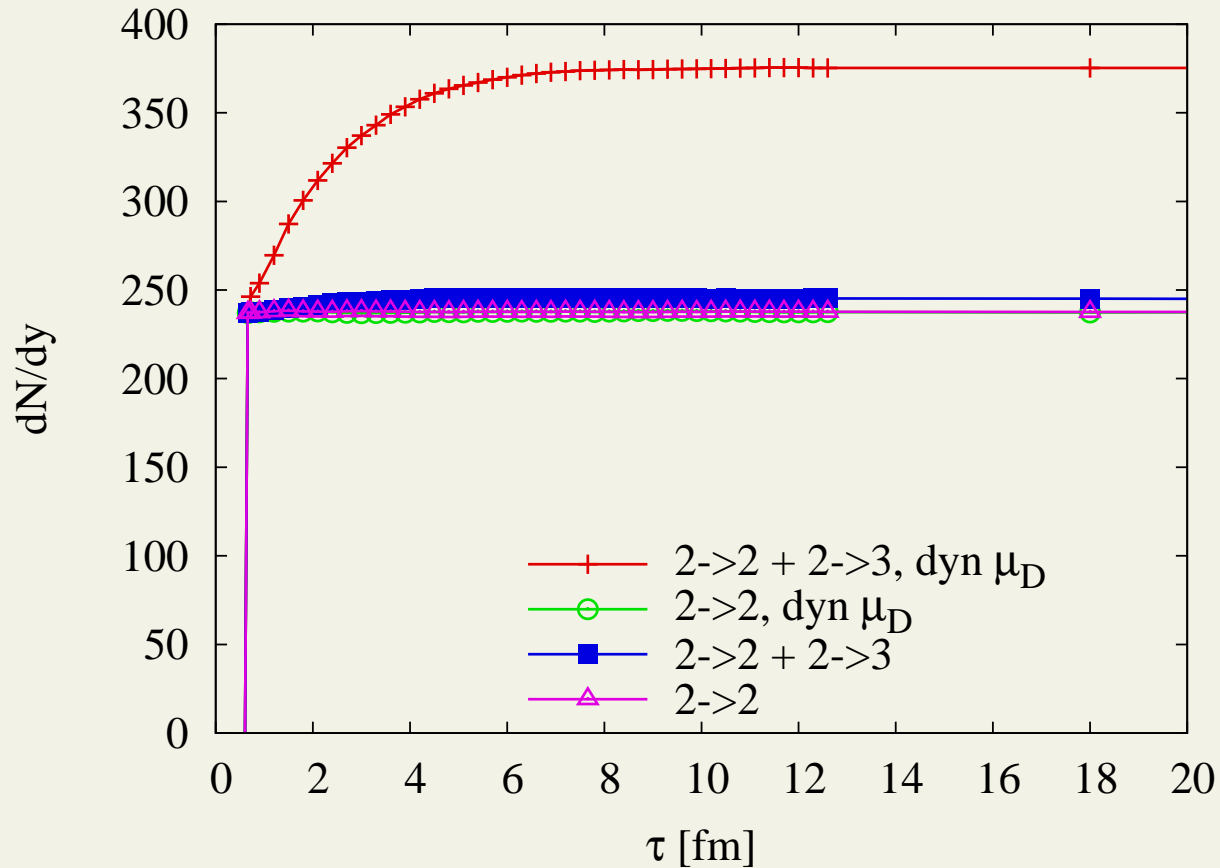


**RHIC Au+Au at  $b = 8$  fm, dynamical  $\mu_D$  vs  $\mu_D = gT_0(\tau_0/\tau)^{1/3}$**



**dynamical  $\mu_D$  helps a lot**

result with dynamically screened  $gg \rightarrow ggg$  is not the final word because



$\Rightarrow$  will need better constrained initconds (chemically better saturated)

## Message:

- hard to reproduce BAMPS results, not clear what they did
- big sensitivity to screening, dynamical  $\mu_D$  looks crucial
- we get significantly smaller thermal rates than BAMPS

# Self-consistent viscous $\delta f$

DM & Zack Wolff (& Qiang Zhang)

# Hydro $\rightarrow$ particles

$$T^{\mu\nu}(x) \equiv \sum_i \int \frac{d^3p}{E} p^\mu p^\nu f_i(p, x)$$

- in local equilibrium (ideal hydro) - “one to one”

$$T_{LR}^{\mu\nu}(x) = \text{diag}(e, p, p, p) \quad \Leftrightarrow \quad f_{eq,i}(x, p) = \frac{g_i}{(2\pi)^3} e^{-p^\mu u_\mu / T}$$

- near local equilibrium (viscous hydro) - “few to many”

$$T^{\mu\nu}(x) = T_{ideal}^{\mu\nu}(x) + \pi^{\mu\nu}(x) \quad \Leftarrow \quad f(x, p) = f_{eq,i}(x, p) + \delta f_i(x, p)$$

common choice - “democratic” Grad ansatz:  $\delta f_i \equiv f_i^{eq} \times \frac{\pi^{\mu\nu}}{2(e+p)} \frac{p_{\mu,i} p_{\nu,i}}{T^2}$

- 2 dissipative effects:** - corrections to equilibrium values  $u^\mu, T, n$   
- corrections to thermal distributions  $f \rightarrow f_0 + \delta f$



## Problem: “democratic Grad” ignores microscopic dynamics

$$\delta f_i \equiv f_i^{eq} \times \frac{\pi^{\mu\nu}}{2(e+p)} \frac{p_{\mu,i} p_{\nu,i}}{T^2}$$

instead of

$$\delta f_i \equiv f_i^{eq} \times \chi_i \left( \frac{\tilde{p}}{T} \right) \pi^{\mu\nu} \frac{p_{\mu,i} p_{\nu,i}}{T^2}$$

→ use instead a nonequilibrium approach: linearized kinetic theory

DM, JPG38 ('11); DM & Wolff, arXiv:1404.7850

$$p^\mu \partial_\mu f_{eq,i} = \sum_j C_{ij} [f_{eq,i}, \delta f_j] + C_{ij} [\delta f_i, f_{eq,j}]$$

$$\delta f_j = \chi_j(p/T) \frac{p^\mu p^\nu \pi_{\mu\nu}}{2(e+p)T^2} f_{eq,j}$$

**democratic:**  $\chi_j = \frac{\eta}{2s}$ ,      **dynamical Grad:**  $\chi_j = C_j \frac{\eta}{2s}$       ( $\mu_B = 0$ )

Dynamics of hadron gas matters  $\Rightarrow$  species dependent correction factors

$\sigma_{ij} = \text{const}$

$\sigma_{MM} : \sigma_{MB} : \sigma_{BB} = 4 : 6 : 9$

	T = 100	120	140	165 MeV
$\pi$	1.08	1.13	1.17	1.21
K	0.89	0.96	1.02	1.08
$\eta$	0.87	0.94	1.00	1.06
$f_0$	0.85	0.92	0.98	1.04
$\rho$	0.80	0.87	0.93	0.99
$\omega$	0.80	0.86	0.93	0.99
$K^*892$	0.77	0.83	0.90	0.96
<b>N</b>	<b>0.76</b>	<b>0.82</b>	<b>0.88</b>	<b>0.94</b>
$\eta'(958)$	0.75	0.82	0.88	0.94
$f_0(980)$	0.75	0.81	0.87	0.93
$a_0(980)$	0.75	0.81	0.87	0.93
$\phi(1020)$	0.74	0.81	0.86	0.92
$\Lambda$	0.72	0.79	0.84	0.90
...				

	T = 100	120	140	165 MeV
$\pi$	1.08	1.15	1.21	1.27
K	0.90	0.98	1.06	1.14
$\eta$	0.88	0.95	1.03	1.12
$f_0$	0.86	0.94	1.01	1.10
$\rho$	0.80	0.88	0.96	1.04
$\omega$	0.80	0.88	0.95	1.04
$K^*892$	0.77	0.85	0.92	1.01
<b>N</b>	<b>0.56</b>	<b>0.62</b>	<b>0.68</b>	<b>0.74</b>
$\eta'(958)$	0.76	0.83	0.91	0.99
$f_0(980)$	0.75	0.83	0.90	0.98
$a_0(980)$	0.75	0.83	0.90	0.98
$\phi(1020)$	0.75	0.82	0.89	0.97
$\Lambda$	0.53	0.59	0.64	0.70
...				

corrections drop with mass even for  $\sigma_{ij} = \text{const}(!)$

one-component system in NR limit:

$$\chi^{Grad} = \frac{5\sqrt{\pi}}{32} \sqrt{\frac{T}{m n \sigma_{TOT}}} \frac{T}{\sigma_{TOT}} \quad \eta_s^{Grad} = \frac{5\sqrt{\pi}}{16} \frac{\sqrt{mT}}{\sigma_{TOT}}$$

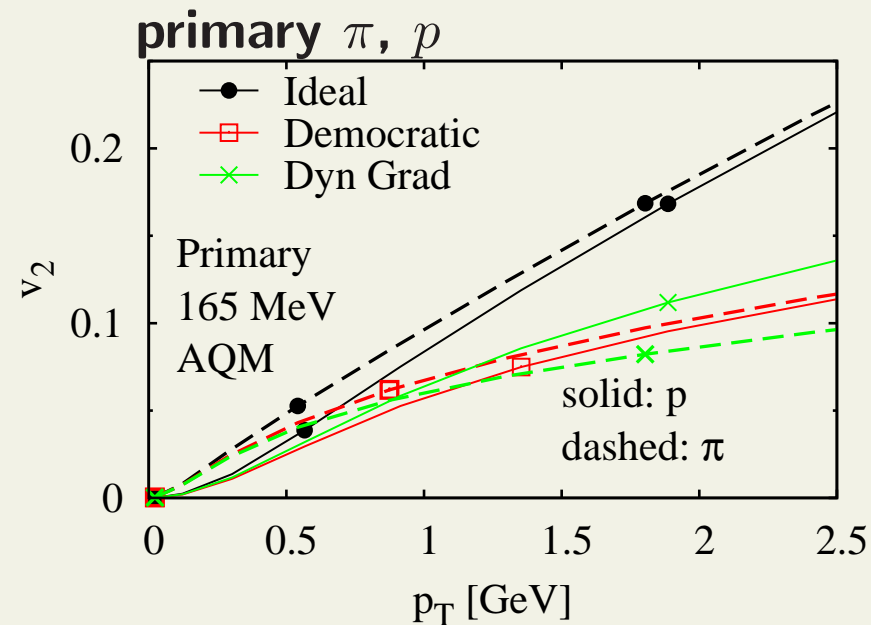
two-component system in NR limit, with  $n_B \ll n_A$ :

$$\chi_A^{Grad} \Big|_{n_B \rightarrow 0} = \frac{5\sqrt{\pi}}{32} \sqrt{\frac{T}{m_A \sigma_{AA} n_A}} \frac{T}{\sigma_{AA} n_A},$$

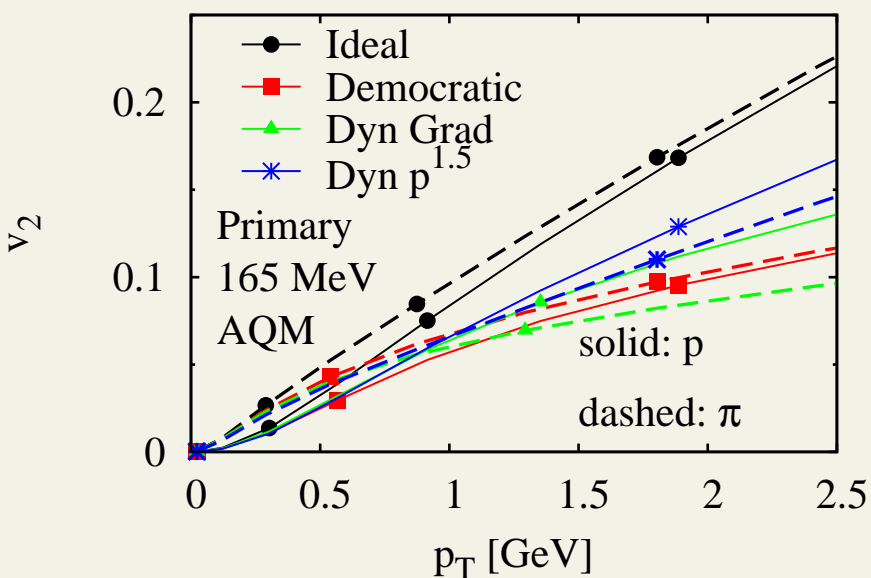
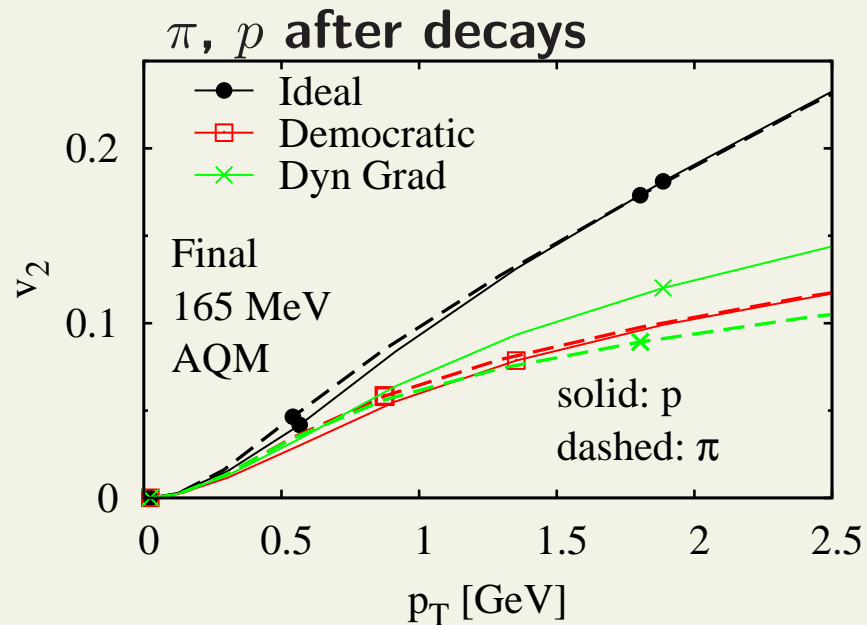
$$\chi_B^{Grad} \Big|_{n_B \rightarrow 0} = \chi_A^{Grad} \frac{3(\mu + 1)^2 \sigma_{AA} + 2\sqrt{2\mu(1 + \mu)} \sigma_{AB}}{\sqrt{2\mu(1 + \mu)} (3 + 5\mu) \sigma_{AB}} \quad \left( \mu = \frac{m_B}{m_A} \right).$$

[DM & Wolff, arXiv:1404.7850]

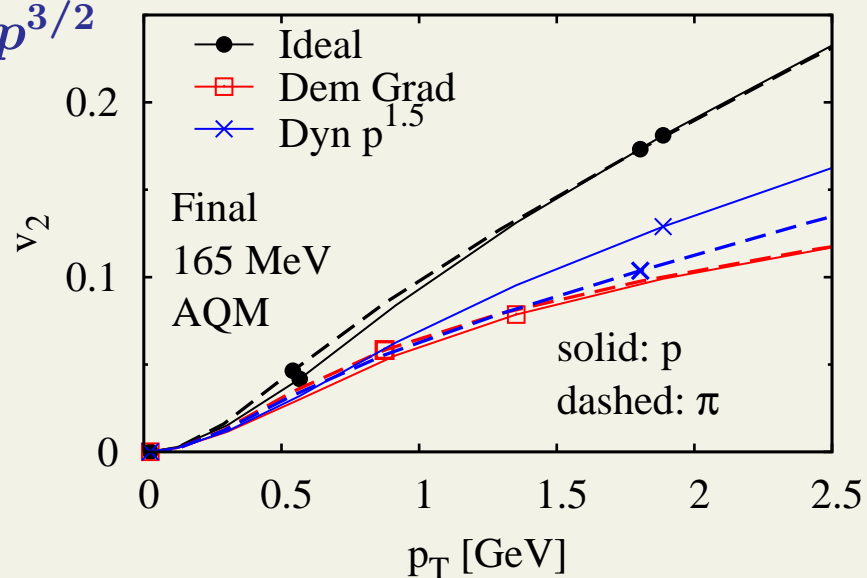
# Affects $v_2$ at RHIC



Grad

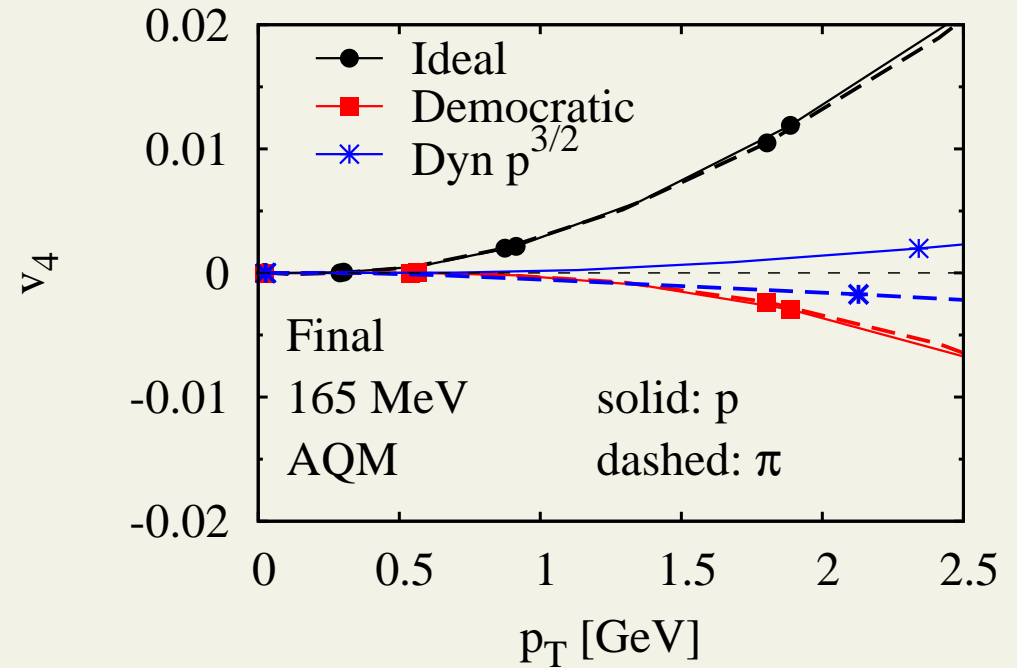
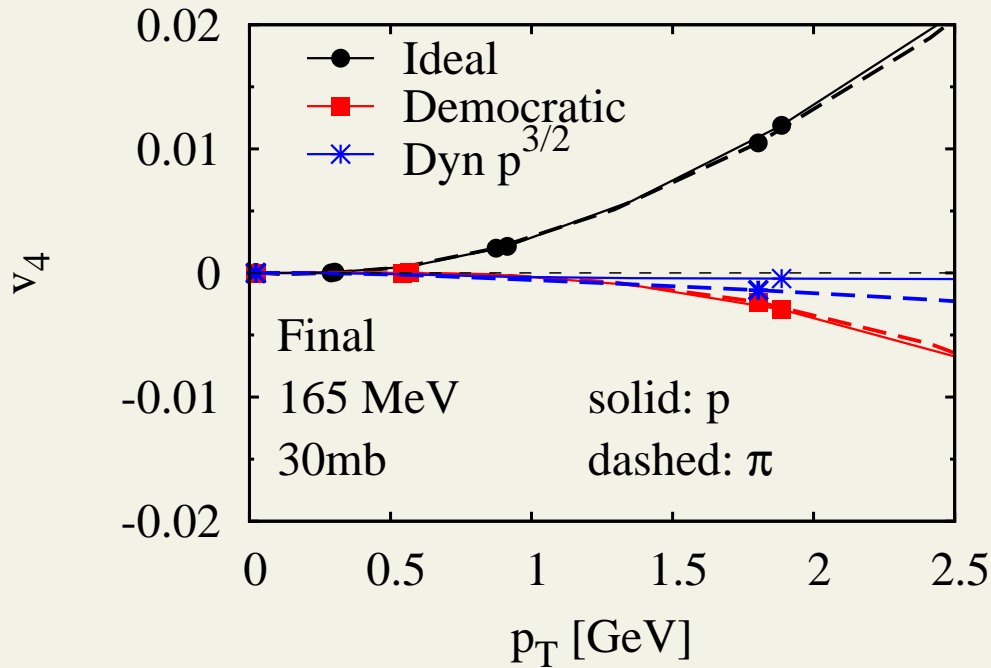


$\delta f \propto p^{3/2}$



# Also affects $v_4$ (and $v_6$ )

$v_4$  after resonance decays, with  $\delta f \propto p^{3/2}$



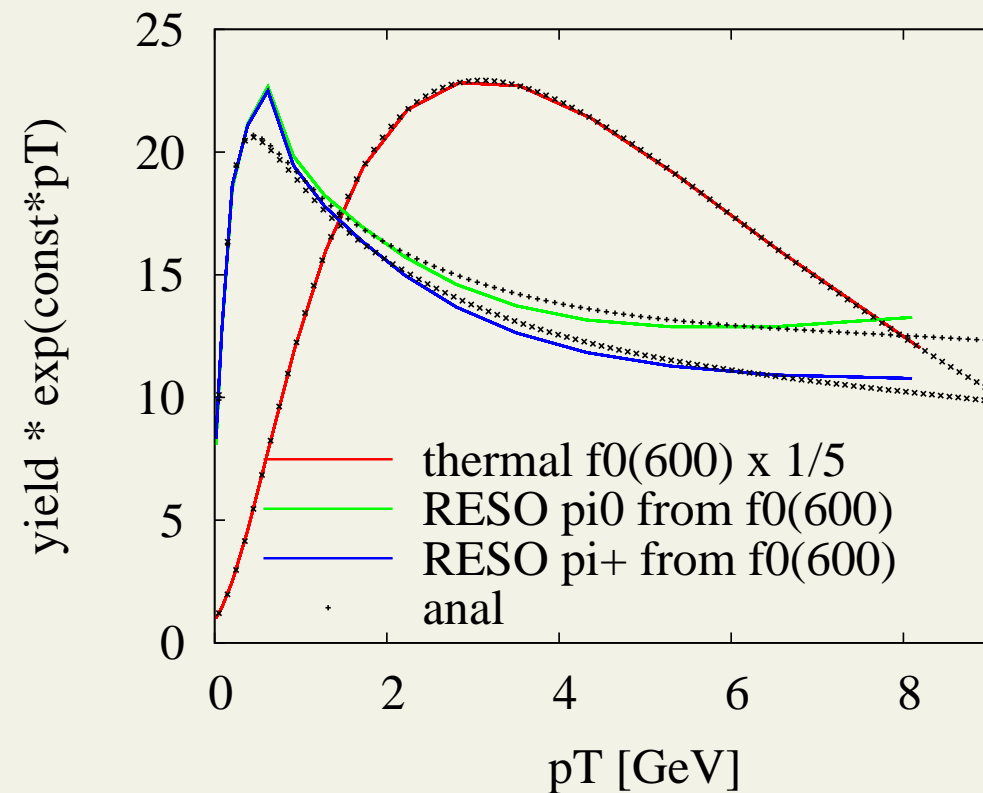
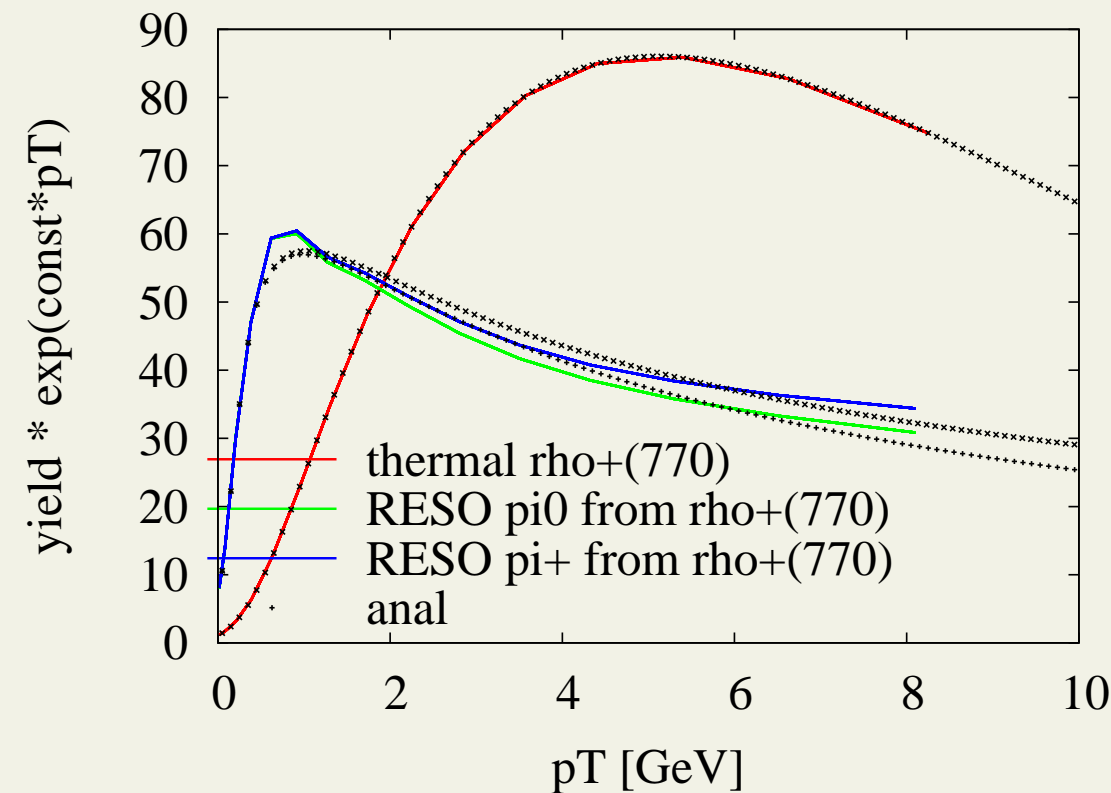
# RESO not super accurate

test of AZHYDRO/RESO, anno 1994 Sollfrank, Heinz et al

daughter spectra for static thermal resonance input, **divide by convenient  $\exp(const \times pT)$  factor**

$\rho \rightarrow \pi\pi$

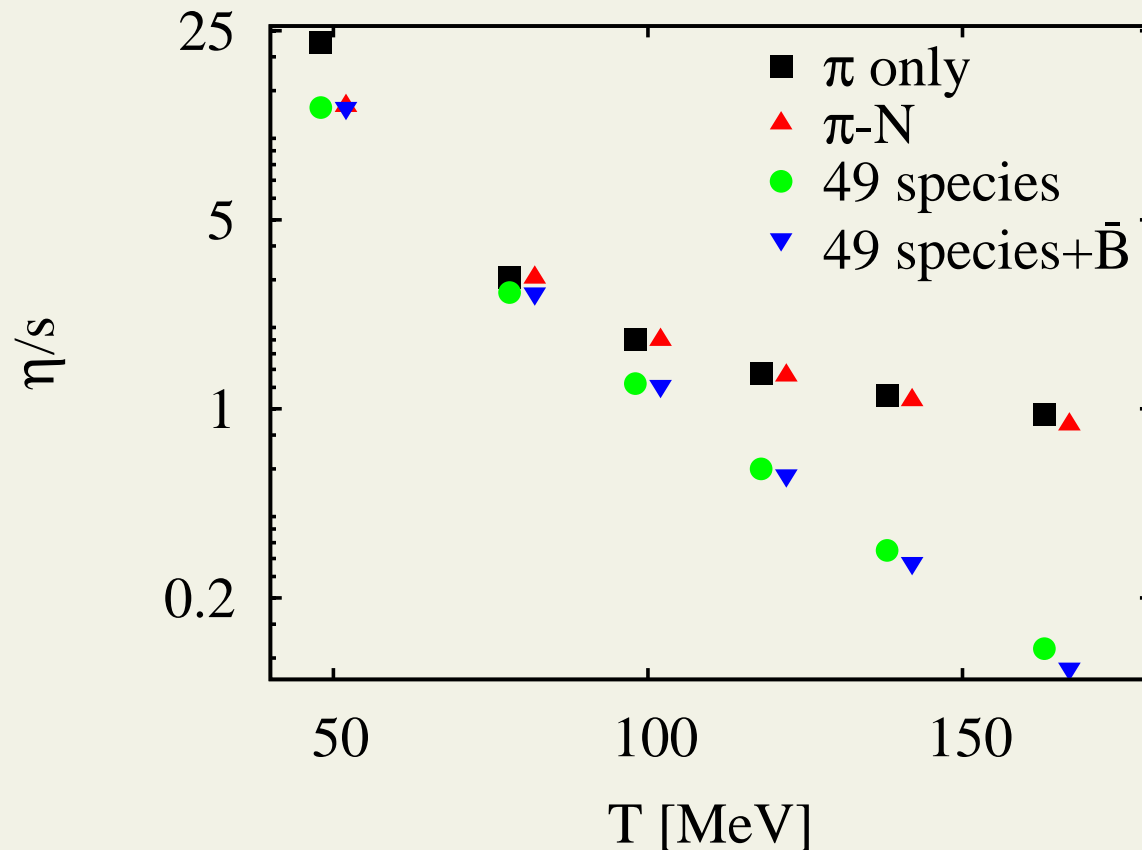
$f_0 \rightarrow \pi\pi$



# Hadron gas viscosity

with realistic  $\sigma(s)$  for  $\pi - \pi$  and  $\pi - N$

or  $\sigma_{MM} = 30$  mb for all hadrons up to  $\sim 1.7$  GeV



## Message:

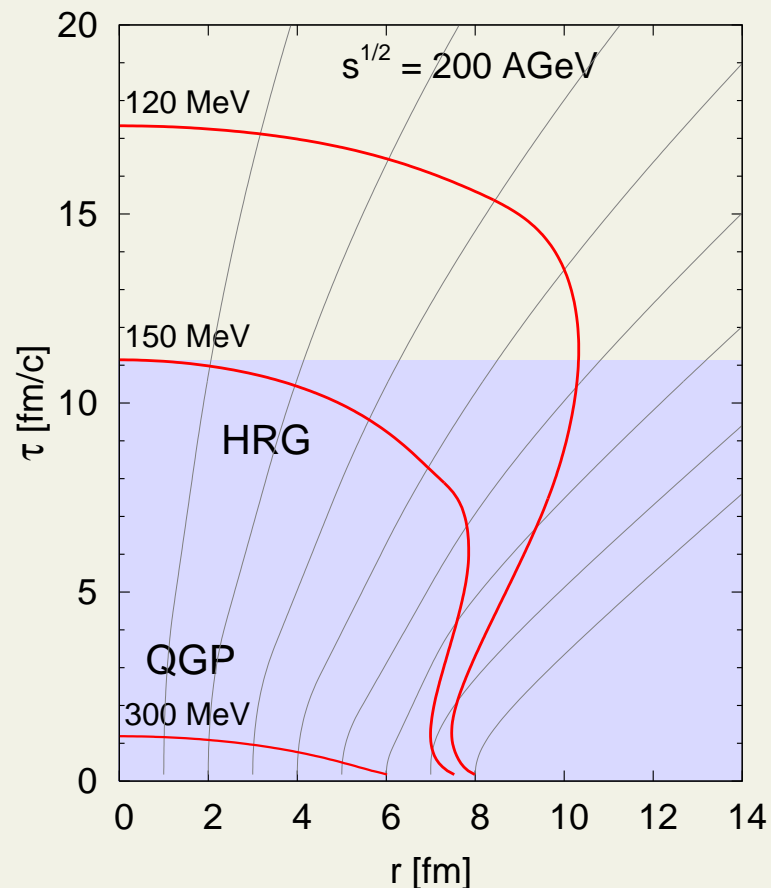
- **viscous corrections depend on hadron species**
- **self-consistent  $\delta f_i$  can be obtained from kinetic theory**
- **dynamical treatment affects harmonic flow**
- **should explore more in hydro and hydro+transport models**
  - **use our tabulated corrections in Appendix F of arXiv:1404.7850**  
**available for  $\delta f \propto p^n$  with  $n = 1, 1.5, 2$**



# Year 5

- **GLV + bulk dynamics (MPC/Eloss)**
  - running  $\alpha_s$ , fluctuations in emission momenta/number
    - ideally merge forces with Columbia
  - heavy quarks with covariance
- **radiative transport MPC/Grid**
  - shear viscosity
  - truly local  $\mu_D(x_T, \eta)$
  - initconds/dynamics that reproduces basic RHIC/LHC observables
- **viscous fluid-particle conversion ( $\delta f$ )**
  - bulk viscous  $\delta f$
  - self-consistent  $\delta f$  in hybrid hydro+transport approach
  - dynamical hydro-transport coupling

**MUST couple hydro and transport dynamically in shaded region below. From pure hydro,  $T = \text{const}$  hypersurface is physically inconsistent (ignores breakdown of hydro outside hypersurface).**



[adapted from Fig. 2 of Huovinen & Ruuskanen, Ann. Rev. Nucl. Part. Sci. 56 ('06)]